CREI
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Discussion Paper No. 23

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経済格差研究センター(CREI)は、大阪市立大学経済学研究科重点研究プロジェクト「経済格差と経済学ー異端・都市下層・アジアの視点からー」（2006~2009 年）の推進のため、研究科内に設置された研究ユニットである。

Fertility, Pensions, and Multiple Equilibria[†]

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Abstract: We show that when the government has a target debt level, multiple equilibria exist because of a complementary relationship between the government and individuals. One is associated with a high fertility rate and a high pension level. The other is associated with a low fertility rate and a low pension level. Furthermore, a strict target level of government debt tends to reduce the low fertility rate. We obtain the same results with consumption taxes. The government should provide adequate security for individuals during their retirement years not to cause a failure of coordination between the government and individuals.

Keywords: Fertility, Pensions, Multiple equilibria.

JEL Classification: J13, H55, G23.

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1. Introduction

The fertility rate in Japan has been decreasing, while the ratio of elderly people to the total population has been increasing. In addition, the Japanese government has attempted to reduce the number of issued government bonds by implementing the Reform of Fiscal Structure Law in 1997, because the government debt in Japan had increased significantly during the 1990s. This led to considerable controversy over the sustainability of the pay-as-you-go (PAYG) social security scheme in an aging society with fewer children and a large public debt.[1]

Should the government reduce the pension level in order not to increase the government debt as the pensionable proportion of the population rises? This paper assesses the possible relationship between the level of pension and the fertility rate when the government has a target for restricting the level of government debt. Pension levels affect lifetime income, which in turn affect the fertility rate. We show that given income taxes or consumption taxes, there exist multiple equilibria between pension levels and fertility rates. If the government chooses to set pensions at a low level, the fertility rate of those dependent on such pensions decreases during their child-rearing years. A low pension level means that individuals cannot count on a reasonable income during their retirement years. Thus, they would seek to have fewer children during their younger years so as to reduce their household living expenses. The government then cannot increase the pension level because of the low level of tax revenue. On the

[1] A decline in the fertility rate and growing public debt are also evident in South Korea and Germany.

other hand, if the government chooses to fix pensions at a higher level, this would tend to increase the fertility rate, as the prospect of larger pensions enables individuals to consider having more children. The government, in due course, obtains more tax revenue, because a higher fertility rate eventually expands the size of the taxpaying labor force relative to the pensionable group. This would justify setting the high pension level.[2] In addition, a lower target level of debt would tend to make the low fertility rate even lower and the low pension level even lower.

We must emphasize the following points. Assuming a balanced government budget, Groezen, Leers, and Meijdam (2003), Groezen and Meijdam (2008), and Hirazawa and Yakita (2009) examined the effects of the PAYG social security scheme on fertility and welfare. As in their models, our model also considers endogenous fertility, in that children are assumed to be consumption goods. We show that the existence of a target level for government debt yields a complementary relationship between the government and individuals. Multiple equilibria then exist in the relationship between the pension level and the fertility rate.[3]

2. Model

We consider a small, open, overlapping-generations model. Individuals

[2] In Japan and some European developed countries, including Italy, Germany, France, and Sweden, the correlation between the fertility rate and the ratio of pension benefit to labor income is 0.694, which is high. The data on the ratio of pension benefit to labor income are taken from Pensions at a Glance, 2009 (OECD). The data describing the fertility rate in 2007 are taken from the Demographic Yearbook (UN) and Vital Statistics (Japan).

[3] Ono (2003) examined fiscal sustainability in his exogenous population growth model. By introducing a target level of government debt, Futagami, Iwaisako and Ohdoi (2008) showed that multiple equilibria exist in the relationship between productive government spending and economic growth.

live in three periods. In the first period, the cost of child rearing is paid for by parents. In the second period, during which individuals are working, they decide how much to consume, how much to save, and how many children to have. They have to pay income taxes during this time. In the third period, they receive a pension and consume the savings. The government collects income taxes and issues bonds used for providing pensions while maintaining a target level of government debt. Firms are assumed to be perfectly competitive. The interest rate which is exogenously given is assumed to be constant. This implies that the wage rate also remains constant.

2.1 Individuals

The utility maximization problem for an individual born in period *t-1* is expressed as:

$$\max_{n_t, c_{1t}, c_{2t+1}} \quad \alpha \ln n_t + \beta \ln c_{1t} + (1-\alpha-\beta)\ln c_{2t+1}, \tag{1}$$

$$s.t. \qquad c_{1t} + \frac{c_{2t+1}}{1+r} + z n_t = (1-\tau)w + \frac{p}{1+r}, \tag{2}$$

where $0 < \alpha, \beta < 1$, and $\alpha + \beta < 1$. n_t is the number of children born in period t, c_{1t} and c_{2t+1} are the consumption levels in periods t and $t+1$, respectively, τ is the income tax rate, p is the pension level, z is the cost of child rearing, r is the interest rate, and w is the wage rate.

The first-order conditions of the utility maximization problem yield:

$$c_{1t} = c_1 = \beta\left[(1-\tau)w + \frac{p}{1+r}\right], \tag{3}$$

$$c_{2t+1} = c_2 = (1-\alpha-\beta)(1+r)\left[(1-\tau)w + \frac{p}{1+r}\right], \tag{4}$$

$$n_t = n = \frac{\alpha}{z}\left[(1-\tau)w + \frac{p}{1+r}\right]. \tag{5}$$

The relationships between consumption in periods *t* and *t+1* and the fertility rate in period *t* are linear because of the log-linear function of utility. The number of children is proportionate to their parents' lifetime income.

2.2 The government

The government finances the pension system by two methods: levying income taxes and issuing bonds. Its budget constraint can be written as:

$$B_{t+1} = (1+r)B_t + N_{t-1}p - N_t\tau w,$$

where B_t is the government debt in period *t*, and N_t is the population born in period *t-1*.

The debt-per-capita dynamics then become:

$$b_{t+1} = \frac{1+r}{n}b_t + \frac{p}{n^2} - \frac{\tau w}{n}, \tag{6}$$

where $b_t \equiv B_t/N_t$, which is public debt per capita, and $N_{t+1}/N_t = N_t/N_{t-1} = n$.

The government is assumed to have a target level of debt per capita, which is represented by $\bar{b} > 0$. By controlling the income tax rate and the pension level, the government tries to satisfy $\bar{b}$. When $p/n^2 - \tau w/n > 0$, which implies a negative primary balance, $(1+r)/n$ must be smaller than unity.[4] Given the initial value, b_0, it converges to $\bar{b}$. This target level must satisfy:

$$\bar{b} = \frac{1+r}{n}\bar{b} + \frac{p}{n^2} - \frac{\tau w}{n}. \tag{7}$$

[4] If $(1+r)/n > 1$ in the case of a positive primary balance, the government debt can converge to a zero value.

3. Multiple equilibria

Figure 1 shows the relationship between the fertility rate and the pension level represented by (5) and (7). Given $\bar{b}$ and τ, (5) implies a linear relationship. A rise in the pension level causes an increase in the fertility rate, because it increases the level of lifetime income. Since higher pensions provide individuals with added security after their retirement, it allows them to have more children. On the other hand, (7) represents a nonlinear relationship between the fertility rate and the pension level. When the fertility rate is low, the government can pay only low pensions because of the low level of tax revenue. A higher pension level becomes possible with a higher fertility rate because of the higher level of tax revenue. That is, it is easier for the government to maintain its target level of debt.

Using (5) and (7), the fertility rate in equilibrium can be represented as:

$$n_l = 2\left[A + \left(A^2 - 4B\right)^{1/2}\right]^{-1}, \tag{8}$$

$$n_h = 2\left[A - \left(A^2 - 4B\right)^{1/2}\right]^{-1}, \tag{9}$$

where $A \equiv \frac{1}{1-\tau}\left(\frac{z}{\alpha w} + \frac{\bar{b}}{w} - \frac{\tau}{1+r}\right)$, and $B = \frac{\bar{b}}{(1+r)(1-\tau)w}$.

Assuming that $A > 0$ and $A^2 - 4B > 0$, there exist multiple equilibria. Eq.(7) implies p_l and p_h which correspond with n_l and n_h, respectively. When the pension level represented by p_l is low, the fertility rate represented by n_l is low. On the other hand, when the pension level represented by p_h is high, the fertility rate represented by n_h is high. If the government can make young individuals believe they will receive the large pension, they will have more children. The government can then provide

higher pensions. However, if the government fails to make them believe this, they will choose to have a lower number of children. This will eventually force the government to provide the low pension level to maintain the budget.

Assuming that $n_l > 1+r$, we have the comparative statistics:

$$\frac{\partial n_h}{\partial \bar{b}} < 0, \quad \frac{\partial n_l}{\partial \bar{b}} > 0, \quad \frac{\partial n_h}{\partial \tau} < 0, \quad \frac{\partial n_l}{\partial \tau} > 0. \tag{10}$$

An increase in the target debt level implies that in Figure 1, the line representing (7) shifts downward. The increase in the target debt level tends to make the low fertility rate higher and the high pension level lower. Thus, if the government sets a low target level to avoid incurring a large debt in an aging society with fewer children, it will reduce the fertility rate. Note that given the income tax rate, a high target level would make it difficult for the government to maintain its budget, and the equilibria might disappear.

An increase in the income tax rate implies that, in Figure 1, while the line representing (5) shifts downward, the line representing (7) also shifts downward. Although an increase in the income tax rate may induce individuals to have fewer children, it also allows the government to pay higher pensions. The latter effect dominates the former effect in our model. Thus, while an increase in the income tax rate decreases the high fertility rate, it increases the low fertility rate.

Here, we present the following proposition.

Proposition 1: *Given $\bar{b}$ and τ, there exist multiple equilibria with certain conditions. One is associated with the high fertility rate and the high pen-*

pension level. The other has the low fertility rate and the low pension level. While a decrease in the target level of government debt decreases the low fertility rate, it increases the high fertility rate.

Next, we consider using consumption taxes to finance the pension system. The budget constraint of an individual given in (2) is rewritten as:

$$(1+\tau_c)\left(c_{1t}+\frac{c_{2t+1}}{1+r}\right)+zn_t=w+\frac{p}{1+r}. \tag{11}$$

The first-order conditions of the utility maximization problem are:

$$c_{1t}=c_1=\frac{\beta}{1+\tau_c}\left(w+\frac{p}{1+r}\right), \tag{12}$$

$$c_{2t+1}=c_2=\frac{1-\alpha-\beta}{1+\tau_c}(1+r)\left(w+\frac{p}{1+r}\right), \tag{13}$$

$$n_t=n=\frac{\alpha}{z}\left(w+\frac{p}{1+r}\right). \tag{14}$$

The budget of the government given in (6) is rewritten as:

$$b_{t+1}=\frac{(1+r)}{n}b_t+\frac{p}{n^2}-\tau_c\left(\frac{c_{1t}}{n}+\frac{c_{2t}}{n^2}\right). \tag{15}$$

By use of (12), (13), (14), and (15), the target level that the government must satisfy becomes:

$$\bar{b}=\frac{1+r}{n}\bar{b}+\frac{p}{n^2}-\frac{\tau_c}{1+\tau_c}\frac{z}{\alpha}\left[\beta+(1-\alpha-\beta)\frac{1+r}{n}\right]. \tag{16}$$

Eqs.(14) and (16) imply the fertility rates that are in equilibrium:

$$n_l=2\left[C+\left(C^2-4D\right)^{1/2}\right]^{-1}, \tag{17}$$

$$n_h=2\left[C-\left(C^2-4D\right)^{1/2}\right]^{-1}, \tag{18}$$

where $C \equiv \frac{1}{w}\left[\frac{z}{\alpha}\left(1-\frac{\tau_c}{1+\tau_c}(1-\alpha-\beta)\right)+\bar{b}\right]$, and $D \equiv \frac{1}{(1+r)w}\left(\frac{\tau_c}{1+\tau_c}\frac{z}{\alpha}\beta+\bar{b}\right)$.

Assuming that $C^2-4D>0$, there exist two possible equilibria. One is associated with the high fertility rate and the high pension level. The other has the low fertility rate and the low pension level.

Assuming that $n_l>1+r$, the comparative statistics are:

$$\frac{\partial n_h}{\partial \bar{b}}<0, \quad \frac{\partial n_l}{\partial \bar{b}}>0, \quad \frac{\partial n_h}{\partial \tau_c}<0, \quad \frac{\partial n_l}{\partial \tau_c}>0. \tag{19}$$

The effects of $\bar{b}$ on the fertility rates are the same as those from using income tax. The government can pay a higher pension level by increasing consumption tax rates. Thus, while an increase in the consumption tax rate decreases the high fertility rate, it increases the low fertility rate.

4. Conclusion

We showed that, when the government has a target debt level, there exist multiple equilibria between the pension level and the fertility rate. If the government fails to provide adequate security for individuals during their retirement years, it would result in a failure of coordination between the government and individuals. This would eventually reduce the proportion of the population that is working and paying taxes. It would then become difficult for the government to make them have more children by increasing pensions.

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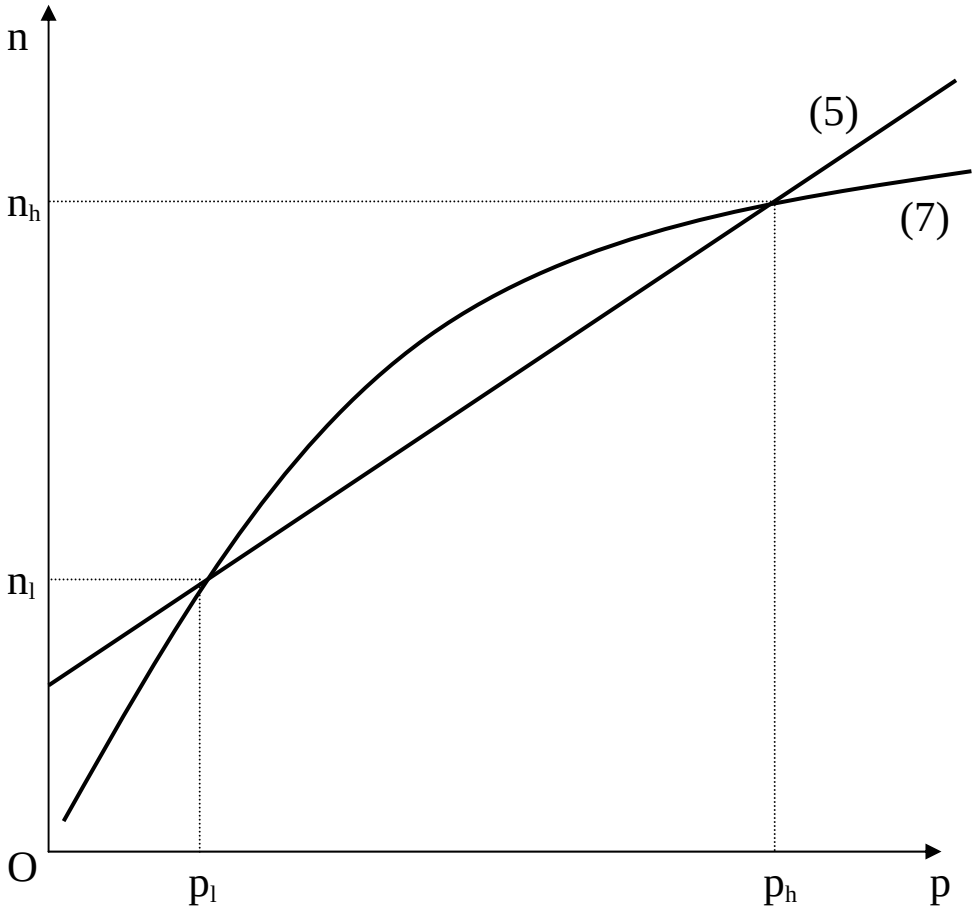

Figure 1. Relationship between pension level and fertility rate