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Care of the Elderly and Public Long-term Care

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Discussion Paper No. 21

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Care of the Elderly and Public Long-term Care*

Yuko Mihara†

Abstract: This paper examines the effect of public long-term care on the macro-economy under a closed two-period overlapping generations economy. We show that the effect of state of health on the steady-state level of capital stock depends on the degree that elderly desires household-produced care service by the young. We also examine the optimal elderly care policy. We show that the government should control both the rate of payroll tax and the rate of self-burden to reach the optimal solution. We also show that the government should set a high rate of self-burden in an economy where the degree of altruism is sufficiently high and the elderly are in good health.

Key words: household-produced care service, market-produced care service, public care system, overlapping generations.

JEL Classification: D64, I18, J14.

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1. Introduction

Most developed countries are experiencing population aging. In the more developed countries, 20 percent of the population is aged 60 years or over and that proportion is projected to reach 33 percent by 2050. In developing countries, in which population aging is less advanced, 8 percent of the population is aged 60 or over but 20 percent of their population is expected to be aged 60 or over by 2050. The factor contributing population aging is both the reduction of fertility and the reduction of mortality at adult ages. In respect of mortality, global life expectancy is estimated to have risen from 58 years to 75 years in 2045-2050. (World Population Prospects: The 2006 Revision). Thus, population aging is not only the problem of developed countries but also it is on the rise around the world. In Japan, where the speed of population aging is the fastest and life expectancy of female is the highest in 2007, at 86 years[1], several problems arise as a result. The most serious problems are a decrease in numbers in the productive labor force, care of the elderly, and the management of the pension scheme for all others. This paper particularly focuses attention on the problem of the care of the elderly.

In Japan, an elder-care insurance scheme was introduced in 2000. As a result, the cost of elder-care insurance is increasing.[2] It is inevitable that both the tax burden and personal expense are growing because population aging leads to the increase of elder-care costs.

The question is how to set the level of both the tax burden and personal expense. Considering this problem, we assume two forms of elder-care service: household-produced care provided by the young using their own time, and a market-produced care service. In this context, the effect of state of health on the steady-state level of capital stock depends on the degree that elderly desires household-produced care service by the young.

To analyze the policy, we develop an overlapping-generations model with public

[1] Japanese Ministry of Health, Labor and Wealth (Abridged Life Tables (2007)).

[2] According to the Japanese Ministry of Health, Labor and Wealth (2006), the cost of the elder-care insurance scheme rose from 3.6 trillion yen in 2000 to 7.1 trillion in 2006.

care system and we showed that the government should set a high rate of self-burden in an economy where the degree of altruism is sufficiently high and the elderly are in good health.

The rest of the paper is organized as follows: Section 2 explains our model without the government; Section 3 analyzes the equilibrium with the government; Section 4 shows the policy of elderly care; and Section 5 states the conclusions of this paper.

2. The model

We consider a closed overlapping-generations economy in which individuals live for two periods of life, working when they are young and retiring when old. All the young are in good health, but the old are in poor health.[3] There are two forms of elderly care service. One is household-produced care service and the other is market-produced care service. In the former, the young donate their time to provide the care. In the latter system, the elderly person buys the care service himself or herself, paying the market price. The economy is closed and homogeneous agents are born in every period. At each period t, all agents have a unit length of time.

2.1 Households

Households have altruism towards parents, and the young take care of their parents by means of spending their time.[4] Thus, we assume that household-produced health technology is:

$$\begin{aligned} h_t &= h[1-l_t] \\ &= 1-l_t \equiv e_t, \end{aligned} \tag{1}$$

where l_t represents the labor supply and e_t represents the time spent caring for the parents.

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[3] Mizushima (2008) examined the relationship the rate of life expectancy and the steady- state level of capital stock assuming that the young supply household-produced care service for the old.

[4] In contrast, the elderly do not look after the young.

The young earn wage income $w_t l_t$ by working for a firm, and spend it for consumption c_t and saving s_t. Then the young face the following budget constraint:

$$w_t l_t = c_t + s_t, \tag{2}$$

When old, they consume part of the proceeds of their saving, d_{t+1}, and the remainder is used for purchasing the market care service $p_{t+1}m_{t+1}$. Then the elderly face the following budget constraint:

$$R_{t+1}s_t = d_{t+1} + p_{t+1}m_{t+1}, \tag{3}$$

where p_{t+1} denotes the price of market care service, m_{t+1} denotes the quantity of purchasing the market-produced care service and R_{t+1} denotes the 1+real interest rate.

We assume that the utility of generation t, U_t, is given as:

$$U_t \equiv u_t + \beta v_{t+1}, \tag{4}$$

where u_t represents the utility of generation t in the working period and u_t depends not only on the consumption of generation t in the working period but also depends on the utility of parents who belong to the retirement period. And v_{t+1} represents the utility of generation t in the retirement period. Thus, we consider a logarithmic utility function

$$u_t \equiv \log c_t + \sigma v_t, \tag{5}$$

$$v_{t+1} \equiv \rho \log d_{t+1} + (1-\rho)[\varepsilon \log e_{t+1} + (1-\varepsilon)\log m_{t+1}], \tag{6}$$

where $\sigma \in (0,1)$ measures the degree of altruism, $\rho \in (0,1)$ denotes the parameter measuring preference for consumption when elderly, so that we can also interpret ρ as state of health.[5] $\varepsilon \in (0,1)$ denotes the parameter measuring preference for household-produced care service.

Thus, each household faces the following utility maximization problem:

$$\max_{c_t, e_t, d_{t+1}, m_{t+1}} \quad U_t = \log c_t + \sigma v_t + \beta[\rho \log d_{t+1} + (1-\rho)[\varepsilon \log e_{t+1} + (1-\varepsilon)\log m_{t+1}]],$$

$$s,t. \qquad w_t l_t = c_t + s_t,$$

$$R_{t+1}s_t = d_{t+1} + p_{t+1}m_{t+1}.$$

[5] The elderly would prefer consumption rather than care service while in good health, but would prefer care service to consumption if in poor health.

Solving this problem, denoting $1/\{1+\sigma\varepsilon(1-\rho)+\beta[\rho+(1-\rho)(1-\varepsilon)]\}$ as Ω, we derive the following:

$$e_t = \Omega\sigma\varepsilon(1-\rho) \equiv e[\rho], \tag{7}$$

$$l_t = \Omega[1+\beta\{\rho+(1-\rho)(1-\varepsilon)\}] \equiv l[\rho], \tag{8}$$

$$s_t = \Omega\beta[\rho+(1-\rho)(1-\varepsilon)]w_t \equiv s[\rho, w_t], \tag{9}$$

$$m_{t+1} = (1-\rho)(1-\varepsilon)R_{t+1}s[\rho, w_t]/\{p_{t+1}[\rho+(1-\rho)(1-\varepsilon)]\}. \tag{10}$$

2.2 Production sector

This section examines the behavior of the production sector. We assume that both the goods sector and the market-produced care service sector are perfectly competitive and all firms practice profit maximization.

Firms that produce the goods face a Cobb-Douglas production function:

$$Y_t = AK_t^{\alpha}\left(L_t^g\right)^{1-\alpha},$$

where $Y_t, K_t, L_t^g, A > 0$ denote the aggregate output of goods, the aggregate capital stock, the labor input of the goods market and a productivity parameter, respectively.[6] The first-order condition for profit maximization is as follows:

$$w_t = (1-\alpha)Ak_t^{\alpha}, \tag{11}$$

$$R_t = \alpha Ak_t^{\alpha-1}, \tag{12}$$

where $k_t \equiv K_t/L_t^g$.

Next, we turn our attention to the care service sector. We assume that their production technology is linear:

$$M_t = L_t^m,$$

where M_t, L_t^m denote the aggregate output of the care service and the labor input of

[6] We assume that the depreciation rate of the capital stock is one.

the care service sector, respectively.[7]

The first-order condition for profit maximization is

$$p_t = w_t. \tag{13}$$

2.3 Dynamics without the public care system

The equilibrium condition for the capital market, the care service market and the labor market are given by $K_{t+1} = s_t N_t$, $M_t = m_t N_{t-1}$, $L_t = L_t^g + L_t^m = \gamma_t L_t + (1-\gamma_t) L_t$, respectively, where $\gamma_t \in (0,1)$ represents the proportion of labor input in the goods market at period t, and we calculate γ_t as follows[8]:

$$\gamma_t = \frac{(1-\alpha)(1-\varepsilon+\varepsilon\rho)}{\rho(1-\alpha)+(1-\rho)(1-\varepsilon)} \equiv \gamma[\rho]. \tag{14}$$

Note that the proportion of labor input into the goods market is constant over each period, and the more the elderly prefer the household-produced care service, the bigger the proportion of labor input into the goods market becomes [from eq.(14), $\partial\gamma[\rho]/\partial\varepsilon > 0$].

Dividing both sides of the equilibrium condition for the capital market by N_t, and using equations (9), (11), (12) and (14), we obtain the equation describing the equilibrium dynamics of k:

$$k_{t+1} = \Gamma A k_t^{\alpha},$$

where $\Gamma \equiv \dfrac{\beta[\rho(1-\alpha)+(1-\varepsilon)(1-\rho)]}{(1+n)[1+\beta(1-\varepsilon+\varepsilon\rho)]}$.

Given the initial condition (K_0, L_0^g) the labor capital ratio converges to the following steady state level:

$$k = (\Gamma A)^{\frac{1}{1-\alpha}} \equiv k[\rho]. \tag{15}$$

Then, let us consider the effect ρ on capital accumulation.

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[7] Following Yoshida and Kenmochi (2005), the care service sector is labor intensive in general; we assume that market care production does not require capital.

[8] See Appendix for derivation.

Proposition 1. *The effect* ρ *on capital accumulation depends on the size of* ε *.*

(i) A higher ρ *leads to a higher steady-state level of capital when* $\varepsilon > \bar{\varepsilon}$ *.*

(ii) Neither a higher ρ *nor a lower* ρ *affect the steady-state level of capital when* $\varepsilon = \bar{\varepsilon}$ *.*

(iii) A higher ρ *leads to a lower steady-state level of capital when* $\varepsilon < \bar{\varepsilon}$ *.*

Proof. Using equation (15), we derive:

$$\frac{\partial k[\rho]}{\partial \rho} \gtrless 0 \leftrightarrow \varepsilon \gtrless \frac{\alpha(1+\beta)}{1+\alpha\beta} \equiv \bar{\varepsilon}.$$

■

An increase in ρ affects the steady state capital accumulation in the following three ways. First is the change in labor supply. A higher ρ leads to an increase in labor supply, and this lowers the steady-state capital accumulation. The second is the change in saving by the young. Because an increase in the labor supply leads to an increase in household's labor income, the young tend to increase their saving. This heightens the steady-state capital accumulation.

The last is the change of $\gamma[\rho]$. A higher ρ leads to an increase in the proportion of labor input into the goods market, and this lowers the steady-state capital accumulation.

3. Model based on public care system

In the following sections, we describe an economy with a public care system. To do so, we introduce the government which runs the public care system.[9] Then, we assume that this system operates by the following rules. First, the elderly must pay the full cost for using a market-produced care service, $p_t m_t N_{t-1}$. The government pays part of it as benefits for the elderly, collecting it as payroll tax from the young. More concretely,

[9] In this paper, we are not concerned with government debt.

denoting the rate of self-burden as $f \in (0,1)$, the government pays $(1-f)p_t m_t N_{t-1}$ as $F_t N_{t-1}$ for the elderly.[10] Thus the government budget constraint at period t is:

$$F_t N_{t-1} = \tau w_t l_t N_t = (1-f)p_t m_t N_{t-1}, \tag{16}$$

where $\tau \in (0,1)$ is the rate of payroll tax.

3.1 Behavior of household with a public care system

Each household faces the following utility maximization problem with a public care system:

$$\max_{c_t, e_t, d_{t+1}, m_{t+1}} \quad U_t = \log c_t + \sigma v_t + \beta[\rho \log d_{t+1} + (1-\rho)[\varepsilon \log e_{t+1} + (1-\varepsilon)\log m_{t+1}]],$$
$$s,t. \quad (1-\tau)w_t l_t = c_t + s_t,$$
$$R_{t+1}s_t = d_{t+1} + p_{t+1}m_{t+1} + F_{t+1}.$$

Solving the problem, we arrive at the following equation:

$$e_t = \Psi \sigma\varepsilon(1-\rho)[w_t(1-\tau) + F_{t+1}/R_{t+1}] \equiv e[\rho], \tag{17}$$

$$l_t = \Psi[(1-\tau)w_t(1+\beta\{\rho + (1-\rho)(1-\varepsilon)\}) - \sigma\varepsilon(1-\rho)F_{t+1}/R_{t+1}], \tag{18}$$

$$s_t = \Psi(1-\tau)w_t[\beta(1-\tau)w_t[\rho + (1-\rho)(1-\varepsilon)] - [1+\sigma\varepsilon(1-\rho)]F_{t+1}/R_{t+1}] \equiv s[\rho, w_t, F_{t+1}], \tag{19}$$

$$m_{t+1} = (1-\rho)(1-\varepsilon)\{R_{t+1}s[\rho, w_t] + F_{t+1}\}/\{p_{t+1}[\rho + (1-\rho)(1-\varepsilon)]\}, \tag{20}$$

where $\Psi \equiv 1/\{(1-\tau)w_t[1+\sigma\varepsilon(1-\rho) + \beta[\rho + (1-\rho)(1-\varepsilon)]]\}$.

3.2 Dynamics with a public care system

In this section, behavior of firms and the equilibrium conditions are the same as section 2, so the equations (11), (12) and (13) still hold.

Calculating the saving in this economy with a public care system is, using equation (16) and (19), we obtain:

$$s_t = \frac{(1-\tau)w_t\beta[\rho + f(1-\rho)(1-\varepsilon)]}{1+\sigma\varepsilon(1-\rho) + \beta[\rho + f(1-\rho)(1-\varepsilon)]}. \tag{21}$$

Then, substituting this equation into (17) and (18), household-produced care

[10] The expense rate is 1 mean that there is no public care system in this economy.

service and labor supply is derived as follows:

$$e_t = \frac{\sigma\varepsilon(1-\rho)}{1+\sigma\varepsilon(1-\rho)+\beta[\rho+f(1-\rho)(1-\varepsilon)]} \equiv e[\rho, f], \tag{22}$$

$$l_t = \frac{1+\beta[\rho+f(1-\rho)(1-\varepsilon)]}{1+\sigma\varepsilon(1-\rho)+\beta[\rho+f(1-\rho)(1-\varepsilon)]} \equiv l[\rho, f]. \tag{23}$$

A higher f leads to a lower $e[\rho, f]$ $\left(\frac{\partial e[\rho, f]}{\partial f} < 0\right)$ and a higher $l[\rho, f]$ $\left(\frac{\partial l[\rho, f]}{\partial f} > 0\right)$. This implies that a higher f creates a higher burden for the elderly; the household then tends to increase labor supply to save more.

Using equations (11), $M_t = m_t N_{t-1}$ and (16), we derive the proportion of labor input in the goods market at period t as follows:

$$\gamma_t = \frac{(1-\alpha)[\rho+f(1-\rho)(1-\varepsilon)]}{\alpha(1-\rho)(1-\varepsilon)+(1-\alpha)[\rho+f(1-\rho)(1-\varepsilon)]} \equiv \gamma[\rho, f]. \tag{24}$$

where $\frac{\partial \gamma[\rho, f]}{\partial \rho} > 0$, $\frac{\partial \gamma[\rho, f]}{\partial f} > 0$.

Substituting equation (11), (21), (23), (24) into $K_{t+1} = s_t N_t$, we obtain the following dynamics of k:

$$k_{t+1} = \Phi(1-\tau)Ak_t^{\alpha}, \tag{25}$$

where $\Phi \equiv \frac{\beta[\alpha(1-\rho)(1-\varepsilon)+(1-\alpha)(\rho+f(1-\rho)(1-\varepsilon))]}{(1+n)[1+\beta(\rho+f(1-\rho)(1-\varepsilon))]}$, and given the initial condition (K_0, L_0^g) the labor capital ratio converges to the following steady-state level k:

$$k = (\Phi(1-\tau)A)^{\frac{1}{1-\alpha}} \equiv k[\rho, \tau, f]. \tag{26}$$

Proposition2. *A higher f (this implies A lower τ) leads to a higher steady-state of capital as long as the government runs the public care system under a balanced* *budget.*

Proof. See Appendix.

■

As long as public care system is operated under a balanced budget, a higher f implies a lower τ. A higher f affects steady-state capital accumulation in the following two ways. As we already know $\frac{\partial l[\rho, f]}{\partial f} > 0$, a higher f leads to an increased saving through increasing labor supply on the part of the young. An increasing saving leads to a higher steady-state capital accumulation. At the same time, as the labor supply increases, this leads to a lower steady-state capital accumulation. Consequently, a higher f leads to higher steady-state capital because the former effect outweighs the latter.

In the rest of this paper, we analyze the elderly care policy. In doing so, we assume social welfare function, denoting δ as the central planner's discount factor, as below:

$$W \equiv \sum_{t=-1}^{\infty} \delta^t \left\{ \log c_t + \left(\frac{\beta + \delta\sigma}{\delta} \right) [\rho \log d_t + (1-\rho)(\varepsilon \log e_t + (1-\varepsilon) \log m_t)] \right\}. \quad (27)$$

We obtain the resource constraints:

$$A(k_t^n)^\alpha (\gamma_t l_t)^{1-\alpha} = c_t + \frac{d_t}{1+n} + (1+n)k_{t+1}^n, \quad (28\text{-}1)$$

$$(1-\gamma_t) l_t = \frac{m_t}{1+n}, \quad (28\text{-}2)$$

where $k_t^n \equiv K_t / N_t$.

Equation (28-1) is an allocation of goods, and equation (28-2) is an allocation of the market care service. Therefore, the social planner maximizes equation (27) under the resource constraints (28-1) and (28-2) with (K_0, d_{-1}) given.

The first-order conditions of the social planner's problem yield:

$$\frac{1}{c_t} = \frac{\rho(\beta + \delta\sigma)(1+n)}{\delta d_t}, \quad \frac{1 - l_t}{c_t} = \frac{\varepsilon(1-\rho)\phi}{(1-\alpha)Ak_t^{\alpha}} \text{ and } \gamma_t l_t = 1 - \left(\frac{\beta + \delta\sigma}{\delta}\right)\frac{\varepsilon(1-\rho)}{(1-\alpha)Ak_t^{\alpha}} c_t.$$

Relationships exist between consumption by the young and consumption by the elderly and the household-produced care service and the labor input toward goods market.

Then, the dynamics of the optimal economy are described by the following difference equations:

$$\frac{c_{t+1}}{c_t} = \frac{\delta\alpha A k_{t+1}^{\alpha-1}}{1+n} \text{ and } (1+n)k_{t+1} = Ak_t^{\alpha} - \psi c_t,$$

where $\psi \equiv \frac{\delta(1-\alpha) + (\beta+\delta\sigma)\left[\rho(1-\alpha) + (1-\rho)(1-\alpha\delta)\right]}{\delta(1-\alpha)} > 0$.

The steady state values of this economy are follows:

$$k = \left(\frac{\delta\alpha A}{1+n}\right)^{\frac{1}{1-\alpha}} \equiv \hat{k},$$

$$e = \frac{\varepsilon(1-\rho)(\beta+\delta\sigma)(1-\alpha\delta)}{\delta(1-\alpha) + (\beta+\delta\sigma)\left[\rho + (1-\rho)(1-\alpha\delta)\right]} \equiv \hat{e}.$$

In the rest of this section, we give our attention to analyzing whether or not an optimal rate of payroll tax and an optimal rate of self-burden exist. First, we obtain the relationship between the rate of payroll-tax and the rate of self-burden by comparing the steady-state capital stock, which is the competitive equilibrium value, and the optimal capital:

$$\tau = \frac{(1-\rho)(1-\varepsilon)\left[\alpha\beta + f\left(\beta(1-\alpha) - \alpha\beta\right)\right] + \beta\rho(1-\alpha) - \alpha(1+\beta\rho)}{\beta\left\{\alpha(1-\rho)(1-\varepsilon) + (1-\alpha)\left[\rho + f(1-\rho)(1-\varepsilon)\right]\right\}} \equiv \hat{\tau}[f]. \quad (29)$$

Next, the comparison between the competitive equilibrium value of household-produced care service and the optimal one yield:

$$f = \frac{\sigma(1+\beta+\sigma) - (\beta+\sigma)\left[1 + \sigma\varepsilon(1-\rho) + \beta\rho\right]}{\beta(\beta+\sigma)(1-\rho)(1-\varepsilon)} \equiv \hat{f}. \quad (30)$$

where we assume $e[\rho,0]=\dfrac{\sigma\varepsilon(1-\rho)}{1+\sigma\varepsilon(1-\rho)+\beta\rho}>\hat{e}$ to be satisfied $\hat{f}>0$.

Note that (30) is independent of the payroll-tax. If the rate of self-burden level is set at (30), the household-produced care service would be coincident with optimal one. Then, when the optimal rate of payroll-tax is set at $\hat{\tau}\left[\hat{f}\right]$, the market solution of the capital stock would be coincident with optimal one.

$$\tau=\frac{\alpha\delta(1+\beta)+(1-\alpha)\rho+(1-\rho)(1-\varepsilon)\left[\alpha+\hat{f}(1-\alpha-\alpha\delta\beta)\right]}{\alpha(1-\rho)(1-\varepsilon)+(1-\alpha)\left[\rho+\hat{f}(1-\rho)(1-\varepsilon)\right]}\equiv\hat{\tau}[\hat{f}].$$

Figure 1 illustrates determination of $\hat{\tau}\left[\hat{f}\right]$. As shown in (34) and (35), to reach the first best solution requires the regulation of both the rate of pay-roll tax and the rate of self-burden.

Let us consider the comparative statistic analysis:

$$\frac{\partial\hat{f}}{\partial\rho}=\sigma^2-\beta-\beta^2\begin{cases}<0 & if \quad \sigma\in(0,\sigma^+)\\ >0 & if \quad \sigma\in(\sigma^+,1)\end{cases}$$

where $\sigma^+\equiv\sqrt{\beta+\beta^2}$.

Here, we have the following proposition with regard to the effect of ρ on $\hat{f}$.

Proposition 3: *If the degree of altruism is sufficiently high, the government can set the rate of self-burden high, and vice versa.*

If σ is sufficiently high, the government can set the rate of self-burden high. The young in an economy with higher σ tend to decrease labor supply and tend to spend more time providing household-produced care service to the elderly. However a higher ρ is equivalent in that there are many elderly who have good health and the demand for goods is high. To achieve a bigger demand for goods, the labor input should be distributed more into the goods market by setting a low rate of self-burden $\left(\dfrac{\partial\gamma[\rho,f]}{\partial f}>0\right)$.

5. Conclusion

In this paper, we analyzed the effects of the household-produced care service and the market-produced care service on macro-economy and the optimal elderly-care policy.

To examine these issues, we constructed a closed overlapping-generations economy; there are two forms for elderly-care service (one is household-produced care service and the other is market-produced care service). Using this model, we showed that the effect of state of health (ρ) on capital accumulation depends on the degree that elderly desires household-produced care service by the young (ε).

In the second part of this paper, we examined the optimal elderly care policy. We showed that to reach the optimal solution, both the rate of pay-roll tax and the rate of self-burden must be controlled. We also showed that if the degree of altruism is sufficiently high and the elderly have good health, the government can set the rate of self-burden high to satisfy consumption demands of the elderly by channeling the labor supply into the goods market.

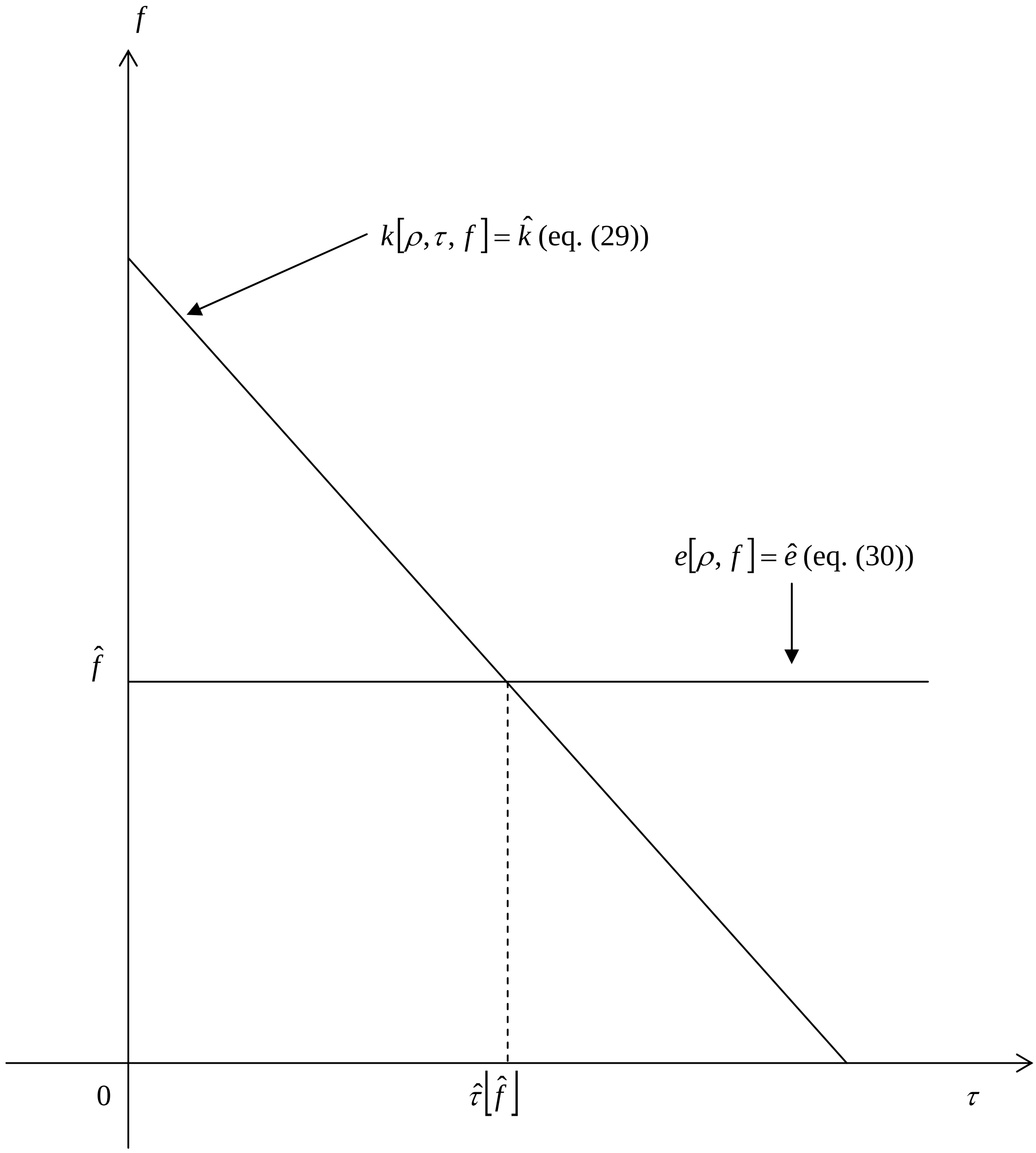

Figure 1: The determination of $\hat{\tau}[\hat{f}]$

Appendix

Derivation of the proportion of labor input into the goods market at period t

Dividing equilibrium condition of capital market and of the care service market into N_t and arranging these equations, we derive the following:

$$(1+n)k_{t+1}\gamma_{t+1}l[\rho] = s_t, \tag{A-1}$$

$$(1+n)(1-\gamma_{t+1})l[\rho] = m_{t+1}. \tag{A-2}$$

Then, by substituting equations (9), (10), (11) and (12) into (A-1) and (A-2) respectively, and using these two equations, we obtain the proportion of labor input into the goods market at period t.

Proof of proposition 2

Inserting equations (11), (21) and (23) into (16) and arranging these equations, we derive the following relationship:

$$\tau = \frac{\alpha(1-f)(1-\rho)(1-\varepsilon)}{\alpha(1-\rho)(1-\varepsilon)+(1-\alpha)[\rho+f(1-\rho)(1-\varepsilon)]} \equiv \hat{\tau}, \tag{A-3}$$

where $\frac{\partial\hat{\tau}}{\partial f} < 0$. Inserting equation (A-3) into (26), we obtain the following:

$$k = \left\{\frac{\beta A[\rho(1-\alpha)+f(1-\rho)(1-\varepsilon)]}{(1+n)[1+\beta(\rho+f(1-\rho)(1-\varepsilon))]}\right\}^{\frac{1}{1-\alpha}}. \tag{A-4}$$

By differentiating (A-4) with respect to f, we obtain proposition 2.

References

Azariadis, Costas (1993). *Intertemporal Macroeconomics*, Blackwell.

Cox ,Donald and Oded Stark (1994). "Intergenerational Transfers and the Demonstration Effect,"*Boston College Working Papers in Economics* 244.

De la Croix, David and Philippe Michel (2002). *A Theory of Economic Growth*, Cambridge University Press.

Japanese Ministry of Health, Labor and Welfare (2006). "Kaigohoken Jigyou Jyoukyou Houkoku (in Japanese)".

Mizushima, Atsue (2008). "Intergenerational Transfers of Time and Public Long-Term Care with an Aging Population," *EUI Working Papers.*

Tabata, Ken (2005). "Population Aging, the Costs of Health Care for the Elderly and Growth," *Journal of Macroeconomics* 27(3) pp.472-493.

United Nations (2006). *World Population Prospects: The 2006 Revision*, http://www.un.org/esa/population/publications/wpp2006/wpp2006.htm.

Yoshida, Masatoshi and Takeshi Kenmochi (2005). "Incidence of Consumption Tax and Tax Treatment of Elderly Care Expenditures," *Public Finance* 54(1-2) pp.57-72.