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November 18, 2009

Discussion Paper No. 19

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The Credit Market, the Rich, and the Poor*

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Abstract: This paper examines the effects of credit availability in very early stages of economic development. Not only the poor but also the rich are assumed to be initially caught in a poverty trap. If the initial wealth level of the rich is higher than a threshold that is lower than the poverty trap, a credit market helps the rich escape from the trap. An improvement in credit availability decreases the threshold. The proportion of the rich also affects the threshold. Furthermore, if the degree of credit availability exceeds a certain level, the poor also can escape from the poverty trap.

Keywords: Degree of Credit Availability, Poverty Trap, Rich, Poor;

JEL Classification: D31, D91, E44.

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1. Introduction

The relationship between finance and economic growth has been investigated by many theoretical and empirical studies. In an extensive survey, Levine (2005) found that better-developed financial systems play an important role in economic growth because they ease external financing constraints facing firms. The relationship between finance and wealth distribution is also important for understanding the process of economic development because wealth distribution influences capital accumulation and resource allocation. Jalilian and Kirkpatrick (2005), Clarke, Xu, and Zou (2006), and Beck, Demirgüc-Kunt, and Levine (2007) found a significant positive effect of financial improvement on the income of the poor.

Assuming credit market imperfection, many theoretical studies investigated the distributions of income and wealth and economic development. In pioneering works, which assumed both an imperfect credit market and nonconvex technology, Banerjee and Newman (1993) and Galor and Zeira (1993) showed persistent inequality. Ghatak and Jiang (2002) also investigated persistent inequality with these assumptions. By replacing technological nonconvexity with a convex bequest function, Moav (2002) reached the same conclusions as Galor and Zeira (1993). Piketty (1997) showed that credit market imperfection is the only factor necessary for the multiplicity of steady states. By considering the equilibrium interest rate, Aghion and Bolton (1997) examined a trickle-down and an efficient allocation of resources. In their model, as more capital is accumulated by the rich, more funds become available to the poor for their investment purposes. Considering a convex bequest function, Galor and Moav (2004) presented a dynamic model to explain both inequality within a country and the process of economic development. They explained that physical capital accumulation by the rich increases the wage income of the poor, and, thereby, helps the poor escape from a poverty trap.[1]

In those studies, it is generally assumed that the poor, but not the rich, are initially caught in a poverty trap. A trickle-down effect can be expected as long as the rich accumulate their wealth. However, not only the poor but also the rich might be caught in a poverty trap in the very early stages of economic development. If a credit market is unavailable in those stages, economic development never occurs because both the rich and poor have been caught in the trap. Can a credit market stimulate economic development by helping the rich escape from the trap? In addition, further macroeconomic development and declines in income inequality and wealth inequality can result when the poor start to accumulate their own wealth. By considering the degree of credit availability, we investigate the effect of improving credit availability on the wealth and incomes of the rich and the poor.

We consider an economy where every individual has an opportunity for investment that is required for production. However, the investment level of the poor

[1]Assuming a convex saving function, Bourguignon (1981) showed that equilibrium characterized by unequal wealth distribution can exist. The relationship between intergenerational mobility and persistent inequality was investigated by assuming credit market imperfections such as those in Freeman (1996), Owen and Wei (1998), Maoz and Moav (1999), Matsuyama (2000), and Mookherjee and Ray (2003).

is smaller than that of the rich because of credit market imperfection. The degree of credit availability is taken into account. Following Moav (2002) and Galor and Moav (2004,2006), a zero bequest is allowed in the utility function. We presume an economy in the very early stages of economic development. While the initial wealth level of the rich is positive, its level is in a poverty trap. The initial wealth level of the poor is zero. Thus, considering the degree of credit availability with the initial condition in which even the rich are caught in the trap, the current paper complements Moav (2002) and Galor and Moav (2004,2006) while a convex bequest function plays a crucial role in yielding the trap.

We show that, if the initial wealth level of the rich is higher than a threshold that is lower than the poverty trap, a credit market can help the rich escape from the trap. That is, a credit market can stimulate economic development even if it is initially impossible for an economy to start to grow. The threshold for the initial wealth decreases by improving credit availability. Given the degree of credit availability, the proportion of the rich must be small to escape from the trap. When credit availability is improved, the income levels of both the rich and the poor increase. However, we show that the poor are caught in the trap unless the degree of credit availability exceeds the threshold.

We emphasize some important points. We clarify two significant effects of a credit market and explain how a credit market helps the rich escape from the trap. The first is the resource allocation effect: resources are more efficiently allocated by an improvement in credit availability. The second is the income distribution effect:

there exists an income stream from borrowers to lenders through interest payments. The poor can borrow small amounts at high interest rates because of the scarcity of credit. Their income level is still inadequate to leave any assets to their children after they have paid their debts. However, the rich who receive payments at high interest rates can rapidly accumulate their wealth. Therefore, a credit market can stimulate economic development by helping the rich escape from the trap even when they are initially caught in the trap.

Second, we show that an improvement in credit availability increases the income levels of both the rich and the poor although, given a low degree of credit availability, it increases wealth inequality between them. The poor cannot start to accumulate their own wealth when the degree of credit availability is low. The investments of the rich and the poor are financed by the wealth of the rich. The improvement in credit availability increases the investment level of the poor and decreases the level for the rich because of the concavity of production technology. The overall incomes of the rich increase because their interest incomes increase. On the other hand, while the incomes of the poor increase, their interest payments also increase. The effect of the improvement on income inequality is then ambiguous.[2] Jalilian and Kirkpatrick (2005) suggested the inverted U-shaped relationship between financial development and income inequality; at a low level of development, financial

[2]In addition, given a low degree of credit availability which is constant, wealth inequality increases as the wealth of the rich accumulates. We also cannot deny the possibility of increasing income inequality with the accumulation of their wealth.

development is positively related to income inequality, but once a threshold level of development is achieved, then the relationship becomes negative.[3] Furthermore, Clarke, Xu, and Zou (2006) found that cross-sectional data did not provide much support for the inverted U-shaped relationship, but some weak support was found with panel data. Our model does not contradict these findings while an improvement in credit availability always alleviates poverty.

Reviewing critically the literature on finance and inequality, Demirg*ü*c-Kunt and Levine (2009) explained substantive gap in the literature. Furthermore, they found that financial development boosts both efficiency and the equality of opportunity. This finding is also implied in our model. Therefore, increasing credit availability should be encouraged to increase the welfare levels of both the rich and the poor even when income inequality and wealth inequality increase at a low degree of credit availability.

The subsequent sections are organized as follows. Section 2 presents a basic model in which there exists no credit market. In Section 3, considering the degree of credit availability, we introduce a credit market. We discuss the dynamics of the credit market in Section 4 and, in Section 5, we conclude with a brief summary and a few remarks.

[3] Greenwood and Javanovic (1990) showed that the distribution effect of financial deepening is temporarily adverse for poor individuals because only a few wealthy individuals have access to higher-return projects. Calibrating a non-steady-state stochastic model, Townsend and Ueda (2006) found a complex relationship among financial deepening, inequality, and growth.

2. Basic Model with no Credit Market

Our model is a closed overlapping-generations economy. If parents decide to leave their bequests, their children receive these bequests in the first period. In the second period, they produce a single type of goods, in which investment is required. Given that there is no credit market, a bequest is used for investment. The output can be used for either consumption or bequest. The population of each generation is assumed to be L, a constant. Individuals are identical except for the bequests that are received from their parents. We assume that the initial numbers of rich and poor are respectively, ηL and $(1-\eta)L$.

Each individual born in period $t-1$ receives a bequest from their parent and spends it just for investment because there is no credit market. That is, we have:

$$e_{it-1} = b_{it-1}, \tag{1}$$

where $i = r, p$. b_{rt-1} and b_{pt-1} are the respective bequest levels of the rich and poor received in period $t-1$, and e_{rt-1} and e_{pt-1} are the respective investments of the rich and the poor in period $t-1$.

In period t, each individual produces goods, subject to the following production function:

$$y_{it} = f(e_{it-1}), \tag{2}$$

where $i = r, p$. y_{rt} and y_{pt} are the respective levels of outputs in period t. We assume that $f'(e_{it-1}) > 0$ and $f"(e_{it-1}) < 0$. We also assume that, even when there is no investment, a constant amount of output can be produced by using a traditional technology, i.e., that $f(0) > 0$.

Individuals divide their income between consumption and bequest. We assume the altruistic bequest motive, i.e., the 'joy of giving'. A zero bequest is allowed because individuals can live with no investment. The utility maximization of an individual born in period $t-1$ is as follows:

$$\max_{c_{it}, b_{it}} \alpha \ln c_{it} + (1-\alpha)\ln(b_{it} + \theta), \tag{3}$$

$$s.t. \quad c_{it} + b_{it} = f(e_{it-1}), \tag{4}$$

where $i = r, p$. We assume that $\theta > 0$, $0 < \alpha < 1$, and $(1-\alpha)f(0) < \alpha\theta$. c_{rt} and c_{pt} are the respective consumption levels of the rich and the poor.

The first-order conditions yield:

$$b_{it} = (1-\alpha)f(e_{it-1}) - \alpha\theta, \quad if \quad (1-\alpha)f(e_{it-1}) > \alpha\theta, \tag{5}$$

$$b_{it} = 0, \quad if \quad (1-\alpha)f(e_{it-1}) \leq \alpha\theta. \tag{6}$$

When the income level is low, the bequest level becomes zero because of θ. As explained in Galor and Moav (2004), the implied bequest function can capture the properties of the Kaldorian–Keynesian saving hypothesis.

Taking (1) into account, the dynamics of investment in the case of (5) become:

$$e_{it} = (1-\alpha)f(e_{it-1}) - \alpha\theta. \tag{7}$$

Curve C in Figure 1 depicts this equation. We consider the case that (7) has two stationary states: e^* and e^{**}. In Figure 1, e^* indicates a stable stationary state, where:

$$(1-\alpha)f'(e^*) < 1. \tag{8}$$

On the other hand, e^{**} is an unstable stationary state, where:

$$(1-\alpha)f'(e^{**}) > 1. \tag{9}$$

We assume that, in the initial period, the rich have a positive level of bequest and the poor have no bequest. The bequest level of the poor remains zero because the initial wealth level of the poor is zero. That is, the poor have been caught in a low-income trap and can never escape from it. The dynamics of the bequest level of the rich depend on the initial value. When the initial wealth level is below the threshold, it decreases and becomes zero after some time. Then, the output level remains:

$$Y = f(0)L. \tag{10}$$

That is, the economy cannot attain economic development. However, the wealth level of the rich would increase and converge to e^* if their initial level exceeded the threshold e^{**}.

3. Model with a Credit Market

Now, suppose that a credit market becomes available. Because borrowing and lending are possible, the income level is not necessarily equal to the output level. The budget constraint of an individual noted in (4) becomes:

$$c_{it} + b_{it} = I_{it}, \tag{11}$$

where $i = r, p$. I_{rt} and I_{pt} are the respective income levels of the rich and the poor in period t.

Equations (5) and (6) are then modified as:

$$b_{it} = (1-\alpha)I_{it} - \alpha\theta, \quad if \quad (1-\alpha)I_{it} > \alpha\theta, \tag{12}$$

$$b_{it} = 0, \quad if \quad (1-\alpha)I_{it} \leq \alpha\theta. \tag{13}$$

An individual determines the optimal investment level to maximize their income by borrowing or lending. While lenders obtain interest earnings on their assets, borrowers have to pay interest on their debts. The income maximization problem of the rich who are the lenders is:

$$\max_{e_{rt-1}} I_{rt} = f(e_{rt-1}) + r_t(b_{rt-1} - e_{rt-1}), \tag{14}$$

where r_t is the gross interest rate in the credit market.

The investment level is determined as:

$$f'(e_{rt-1}) = r_t. \tag{15}$$

Equation (12) is rewritten as:

$$b_{rt} = (1-\alpha)[f(e_{rt-1}) + f'(e_{rt-1})(b_{rt-1} - e_{rt-1})] - \alpha\theta. \tag{16}$$

On the other hand, the poor who have no wealth become the borrowers. Because the lenders to individuals must have positive costs of keeping track of each borrower, the individual must borrow at a rate higher than r_t. They solve:

$$\max_{e_{pt-1}} I_{pt} = f(e_{pt-1}) - \delta r_t e_{pt-1}, \tag{17}$$

where $\delta \geq 1$.

The investment level of the poor is lower than that of the rich because of an imperfect credit market:

$$f'(e_{pt-1}) = \delta r_t. \tag{18}$$

The degree of credit availability is measured by δ. While, for simplicity, the multiplicative form δr_t is assumed, borrowers must pay higher interests as δ rises. Furthermore, given the interest rate, the cost of keeping track increases with an increase in the investment level. An improvement in credit availability is represented by a decrease in δ. δ takes a value of unity in the case of a perfect credit market in which individuals face the same interest rate regardless of whether they are borrowers or lenders.

4. Dynamics

4.1 Regime I: $b_{pt} = 0$

We now consider a situation where the wealth level of the poor remains zero. This regime is termed regime I. The credit market equilibrium is given by:

$$\eta(b_{rt} - e_{rt}) = (1 - \eta)e_{pt}. \tag{19}$$

The investment of the rich and the poor are financed by the wealth of the rich only.

Using (15) and (18), the relationship between the investment levels e_{rt-1} and e_{pt-1} becomes:

$$\frac{f'(e_{pt-1})}{f'(e_{rt-1})} = \delta.$$

For simplicity, the poor's investment level is assumed to be proportionate to that of the rich:

$$e_{pt} = g e_{rt}, \tag{20}$$

where $g \leq 1$.

The relationship between δ and g is negative. While an increase in g implies an improvement in credit availability, the difference in investment levels between the rich and the poor becomes small. If the credit market is perfect, g takes the value of unity. We have $g = \delta^{-1/(1-\gamma)}$ when the production function is specified as:

$$f(e_{it}) = a_0 + a_1 e_{it}^{\gamma}, \tag{21}$$

where $i = r, p$, $a_0, a_1 > 0$, and $0 < \gamma < 1$.

Using (16), (19), and (20), the dynamics of the investment level can be represented as:

$$e_{rt} = \frac{1-\alpha}{\eta + (1-\eta)g}[\eta f(e_{rt-1}) + (1-\eta) g f'(e_{rt-1}) e_{rt-1}] - \frac{\eta\alpha\theta}{\eta + (1-\eta)g}. \tag{22}$$

The above (22) is drawn as curve D in Figure 2. We assume that curve D is strictly concave, or, equivalently, $\partial I_{rt}/\partial e_{rt-1} > 0$ and $\partial^2 I_{rt}/(\partial e_{rt-1})^2 < 0$. We also assume that (22) takes a negative value when e_{rt-1} approaches zero. In Figure 2, e^D indicates a stable stationary state, and e^{DD} is an unstable stationary state. Curve D approaches curve C as g decreases.

Now, we derive the following lemmas:

Lemma 1: $e^{DD} < e^{**} < e^D < e^*$.

Proof: Suppose that $e_{rt-1} = e^*$ in (22). Then, taking into account (8) and $e^* = (1-\alpha)f(e^*) - \alpha\theta$, we can rewrite (22) as follows:

$$\begin{aligned} e_{rt} &= \frac{\eta}{\eta + (1-\eta)g}[(1-\alpha)f(e^*) - \alpha\theta] + \frac{(1-\eta)g}{\eta + (1-\eta)g}(1-\alpha)f'(e^*)e^* \\ &= \frac{\eta}{\eta + (1-\eta)g}e^* + \frac{(1-\eta)g}{\eta + (1-\eta)g}(1-\alpha)f'(e^*)e^* < e^*. \end{aligned} \tag{23}$$

This implies that curve D at e^* is below the 45-degree line.

On the other hand, suppose that $e_{rt-1} = e^{**}$ in (22). Then, noting (9) and $e^{**} = (1-\alpha)f(e^{**}) - \alpha\theta$, we obtain:

$$e_{rt} = \frac{\eta}{\eta + (1-\eta)g}e^{**} + \frac{(1-\eta)g}{\eta + (1-\eta)g}(1-\alpha)f'(e^{**})e^{**} > e^{**}. \tag{24}$$

This implies that curve D at e^{**} is above the 45-degree line.

Therefore, the point at which curve D intersects the 45-degree line exists between e^{**} and e^*, i.e., $e^{**} < e^D < e^*$. Moreover, because the right-hand side of (22) is negative where $e_{rt-1} \to 0$, curve D intersects the 45-degree line at a point located between 0 and e^{**}, i.e., we have $0 < e^{DD} < e^{**}$. ♢

Lemma 2: *Define $\hat{e}$ as an investment that satisfies:*

$$(1-\alpha)[f(\hat{e}) - f'(\hat{e})\hat{e}] - \alpha\theta = 0. \tag{25}$$

Curves C and D intersect at the point where $e_{rt-1} = \hat{e}$ regardless of the degree of credit market imperfection. In addition, we have $e^{**} < \hat{e} < e^D$.

Proof: See Appendix A. ♢

We consider how an improvement in credit availability affects the investment levels of stationary states. The partial derivative of the investment level with respect to g can be written as:

$$\frac{\partial e}{\partial g} = -\frac{[\eta(1-\eta)/(\eta+(1-\eta)g)^2]\{(1-\alpha)[f(e)-f'(e)e]-\alpha\theta\}}{1-[(1-\alpha)/(\eta+(1-\eta)g)]\{[\eta+(1-\eta)g]f'(e)+(1-\eta)gf"(e)e\}}.$$

The numerator of this equation takes a value of zero at $e = \hat{e}$ because of (25). $f(e) - f'(e)e$ is an increasing function with respect to e. Thus, the numerator takes a negative value at $e = e^{DD}$ whereas it takes a positive value at $e = e^D$. The denominator, on the other hand, measures the difference between the slopes of the 45-degree line and curve D. Thus, the denominator takes a negative value at $e = e^{DD}$ whereas it takes a positive value at $e = e^D$.

The comparative statistics with respect to g are then:

$$\frac{\partial e^{DD}}{\partial g} < 0 \quad and \quad \frac{\partial e^D}{\partial g} < 0. \tag{26}$$

An improvement in credit availability decreases the investment levels of the rich on the stationary states.

The effect of the improvement on the wealth level of the rich, on the other hand, can be written as:

$$\frac{\partial b_r}{\partial g} =$$

$$\frac{1-\eta}{\eta[\eta+(1-\eta)g]}\frac{-(1-\alpha)e(1-\eta)gf"(e)e}{1-[(1-\alpha)/(\eta+(1-\eta)g)]\{[\eta+(1-\eta)g]f'(e)+(1-\eta)gf"(e)e\}}.$$

We then have $\partial b_r/\partial g < 0$ at $e = e^{DD}$ and $\partial b_r/\partial g > 0$ at $e = e^D$. That is, the wealth level of the high-level stationary state increases while its level on the low-level stationary state decreases. This implies that the improvement increases the investment level of the poor on the high-level stationary state, i.e., $\partial g e^D/\partial g > 0$

because of $\eta b_r = \eta e + (1-\eta)ge$.[4]

In regime I, the wealth of the rich is divided into the investments by the rich and investments by the poor. When credit availability is improved, their wealth is more efficiently allocated between the rich and the poor. The concavity of production technology implies that the investment level of the rich decreases while the level for the poor increases. The income and wealth levels of the rich on the high-level stationary state increase because of an increase in the interest payments from the poor to the rich. The income level of the poor on the stationary state also increases because of an increase in their investments. However, given that the wealth level of the poor is zero, the improvement increases wealth inequality between the rich and the poor.

The effect of the improvement on income gap can be written as:

$$\frac{\partial(I_r - I_p)}{\partial g} = [f'(e) + f"(e)\frac{1-\eta}{\eta}ge]\frac{\partial e}{\partial g} + [f'(e)\frac{1-\eta}{\eta} + f"(ge)ge]\frac{\partial ge}{\partial g}.$$

The sign of this equation depends on parameters.[5] We focus on the high-level stationary state. While the income level of the poor increases, the interest payments also increase. The income level of the rich also increases because of the receipt of interest payments. The improvement then has an ambiguous effect on income

[4]In regressions, the ratio of private credit to GDP is generally used as a proxy for financial development. Our model implies that, given the wealth level, an improvement in credit availability increases private credit that is equal to the investments of the poor.

[5]The effect of the improvement on the Gini coefficient also remains ambiguous. The coefficient evaluated at $1-\eta$ can be represented as $(1-\eta)I_p/[\eta I_r + (1-\eta)I_p]$.

inequality between the rich and the poor.

Let us assume that $0 < b_{r,-1} < e^{**}$, i.e., that not only the poor but also the rich are caught in a poverty trap. We first investigate the effect of a credit market on the rich. If a credit market is unavailable, the rich also have been caught in the trap and their wealth level becomes zero after some time. However, when a credit market is available, it is possible for the rich to escape from the trap depending on their initial wealth level. The rich can accumulate their wealth if the initial investment level is greater than e^{DD}, which is lower than the poverty trap threshold:

$$e^{DD} < e_{r,-1} = \frac{\eta}{\eta + (1-\eta)g} b_{r,-1} < b_{r,-1} < e^{**}. \tag{27}$$

That is, if the initial wealth level of the rich exceeds $e^{DD}[\eta/(\eta + (1-\eta)g)]^{-1}$, the rich can increase their wealth and, thereby, economic development can start.

$\partial b_r/\partial g < 0$ holds at $e = e^{DD}$. Note that $b_r = e^{DD}[\eta/(\eta+(1-\eta)g)]^{-1}$. Therefore, it becomes easy for the rich to escape from the trap by improving credit availability even when their initial wealth level is low. In addition, we have to examine carefully condition (27) with respect to the proportion of the rich to the total population. While a high ratio of the rich implies a large value $\eta/[\eta + (1-\eta)g]$, e^{DD} is also affected by η. An increase in η increases the investment level e^{DD} because it implies an increase of the wealth holders. Within the range in which $0 < e_{rt-1} < e^{**}$, we define $H(e_{rt-1}|\eta)$ as follows:

$$H(e_{rt-1}|\eta) \equiv \frac{1-\alpha}{\eta + (1-\eta)g}[\eta f(e_{rt-1}) + (1-\eta)gf'(e_{rt-1})e_{rt-1}] - \frac{\eta\alpha\theta}{\eta + (1-\eta)g} - e_{rt-1}. \tag{28}$$

$H(e_{rt-1}|\eta)$ represents the vertical distance between curve D and the 45-degree line in Figure 2. As shown in Figure 2, the correspondences between $H(e_{rt-1}|\eta)$ and e_{rt-1} are as follows:

$$H(e_{rt-1}|\eta) < 0 \Leftrightarrow 0 < e_{rt-1} < e^{DD} \quad and \quad H(e_{rt-1}|\eta) > 0 \Leftrightarrow e^{DD} < e_{rt-1} < e^{**}.$$

By using a small positive value ϵ $(0 < \epsilon < 1)$, we express the initial wealth level of the rich as $b_{r,-1} = \epsilon e^{**}$. Accordingly, at credit market equilibrium, the investment level is:

$$e_{r,-1} = \frac{\eta}{\eta + (1-\eta)g} b_{r,-1} = \frac{\eta}{\eta + (1-\eta)g} \epsilon e^{**}. \tag{29}$$

Let us consider $H(e_{r,-1}|\eta)$ in which given the production function (21), $e_{r,-1}$ is evaluated at (29). While $H(e_{r,-1}|\eta = 0)$ takes a value of zero, $H(e_{r,-1}|\eta = 1)$ takes a negative value. Moreover, $\partial H(\cdot)/\partial \eta$ takes an infinite value at $\eta = 0$. This implies that $H(\cdot)$ is an increasing function when the proportion of the rich takes a value close to zero. Therefore, we can assure that (27) holds when the ratio of the rich is small.

Proposition 1: *If the initial wealth level of the rich is higher than the threshold written in (27), a credit market can help those caught in a poverty trap escape from the trap. An improvement in credit availability decreases the threshold. While the proportion of the rich also affects the threshold, given (21), the ratio of the rich must be small.*

An intuitive explanation of the above result is as follows. In very early stages of economic development, many individuals have no wealth to invest. A few individuals who have wealth exist. However, investing all their wealth in their own production cannot yield enough income to escape from the low-income trap, because production is subject to diminishing returns. If a credit market is available, the relatively rich individuals lend a part of their wealth to the poor individuals. The payment of interest by the poor would bring the rich enough income to increase their wealth. An individual who belongs to the rich can receive a large amount of interest payments when the proportion of the rich is small. The rich then can escape from the trap.

A question of interest is whether it is always possible for the poor to escape from the trap when the rich can accumulate their wealth. If the investment level of the poor exceeds $\hat{e}$ until the investment level of the rich converges to e^D, the poor can start to accumulate their own wealth. This implies that the poor still cannot escape from the trap if $ge^D < \hat{e}$ holds. An improvement in credit availability increases the income of the poor. However, the poor cannot always escape from the trap in an economy with an imperfect credit market even if the rich continue to accumulate their wealth. While the improvement increases ge^D, there exists g that satisfies $\tilde{g}e^D = \hat{e}$ because of the inequality $\hat{e} < e^D$. Therefore, the poor can start to accumulate their own wealth if the following condition holds:

$$g > \tilde{g}. \tag{30}$$

Proposition 2: *Provided the rich can accumulate their wealth, if the degree of credit availability satisfies (30), the poor who initially have no wealth can start to accumulate their own wealth sooner or later.*

If the degree of credit availability is insufficient, the poor cannot start to accumulate their own wealth, given the burden of paying their debts. Note that the utility level of the poor increases because of an increase in their income level. In addition, the income stream from the poor to the rich makes economic development possible.

If (30) does not hold, i.e., if the poor cannot start to accumulate their wealth, the investment level of the rich converges to e^D. The long-run output is:

$$Y = f(e^D)\eta L + f(ge^D)(1-\eta)L. \tag{31}$$

When both the rich and the poor are initially caught in a poverty trap, the long-run output level of an economy with no credit market is represented by (10). On the other hand, both the investment levels of the rich and the poor are positive because of credit availability. Therefore, the output level of the economy is greater than that with no credit market. Furthermore, an improvement in credit availability increases the output level because an increase in $f(ge^D)(1-\eta)$ is greater than a decrease in $f(e^D)\eta$.[6]

We now consider a redistribution policy. We consider that the amount T is deducted from the wealth of the rich and is given to the poor. This redistribution policy does not affect the investment level at that period because we have:

$$[\eta + (1-\eta)g]e_{rt-1} = \eta(b_{rt-1} - T) + (1-\eta)\eta T/(1-\eta) = \eta b_{rt-1}.$$

However, while deducting T from b_{rt-1} reduces b_{rt}, adding $\eta T/(1-\eta)$ to b_{pt-1} does

[6] Note that $\partial[\eta e^D + (1-\eta)ge^D]/\partial g > 0$.

not increase b_{pt} because the wealth of the poor is, again, equal to zero. Consequently, the investment level at the next period declines. Therefore, in regime I, this redistribution policy is ineffective in helping the poor escape from the trap.

4.2 Regime II: $b_{pt} > 0$

We assume that (27) and (30) hold. When the interest rate is sufficiently low because of the wealth accumulation by the rich and the income level of the poor is sufficiently high, the poor begin to accumulate their own wealth. This regime is termed regime II. We assume that credit availability improves because of a positive wealth level of the poor. For simplicity, a perfect credit market, i.e., $g = 1$ is assumed.[7]

Because individuals face the same interest rate regardless of whether they are borrowers and lenders, the investment level is the same between them:

$$f'(e_{t-1}) = r_t. \tag{32}$$

The dynamics of the bequest levels of the rich and the poor can be represented as:

$$b_{it} = (1-\alpha)[f(e_{t-1}) + f'(e_{t-1})(b_{it-1} - e_{t-1})] - \alpha\theta, \tag{33}$$

where $i = r, p$.

Credit market equilibrium in period t requires:

$$\eta(b_{rt} - e_t) = (1-\eta)(e_t - b_{pt}). \tag{34}$$

[7]The accumulation of wealth by the rich and the poor would be mutually dependent in the case of an imperfect credit market. The dynamics of their wealth levels then would become complicated.

Using (33), the equilibrium condition of the credit market can be rewritten as:

$$\eta\{(1-\alpha)[f(e_{t-1})+f'(e_{t-1})(b_{rt-1}-e_{t-1})]-\alpha\theta\}$$

$$+(1-\eta)\{(1-\alpha)[f(e_{t-1})+f'(e_{t-1})(b_{pt-1}-e_{t-1})]-\alpha\theta\}=e_t.$$

This equation then turns out to be:

$$e_t=(1-\alpha)f(e_{t-1})-\alpha\theta. \tag{35}$$

This is identical to (7), which represents the dynamics of the investment level of the rich when there is no credit market. In Figure 2, (35) is drawn as curve C.

Suppose that $e=e^*$. Then, taking into account $e^*=(1-\alpha)f(e^*)-\alpha\theta$, (33) becomes:

$$b_{it}=(1-\alpha)f'(e^*)(b_{it-1}-e^*)+e^*. \tag{36}$$

This implies that $b_i=e^*$ when $b_{it}=b_{it-1}$. We then have $b_r=b_p=e^*$ in the stationary state. Both wealth inequality and income inequality between the rich and the poor disappear if the economy reaches the stationary state.

The wealth level of the poor remains at zero until their investment level exceeds $\hat{e}$. The wealth level of the rich, on the other hand, continues to increase. The poor start to accumulate their own wealth when their investment level exceeds $\hat{e}$. In this case, the income of the poor can become sufficiently high to help them escape from the trap until the economy converges to the stationary state. The income and wealth levels of both the rich and the poor then continue to increase. Although wealth inequality between the rich and the poor always increases in regime I, it turns to decrease in regime II. Both wealth inequality and income inequality eventually disappear because of the same investment levels of the rich and the poor.

The long-run output is:

$$Y = f(e^*)L. \tag{37}$$

When the poor also accumulate their wealth, the investment of the poor is financed not only by the wealth of the rich but also by their own wealth. The investment level of the rich then increases compared with the case that the poor cannot have their own wealth. The investment level of the poor also increases. Therefore, (37) is greater than (31).

5. Conclusions

We considered an economy in the very early stages of economic development where, not only the poor, but also the rich are initially caught in a poverty trap. We found that a credit market can help the rich escape from the trap if the initial wealth level of the rich is higher than a threshold that is lower than a poverty trap. It becomes easy for the rich to escape from the trap as credit availability improves. Given the degree of credit availability, the proportion of the rich to the total population must be small. Furthermore, it is difficult to help the poor escape from the trap by using a redistribution policy in an economy with inadequate credit availability. If the degree of credit availability exceeds a certain level, the poor also can accumulate their own wealth.

Improving the credit availability improves the welfare levels of both the rich and the poor. However, we should remember that there exist thresholds in the degree of credit availability to help the rich and the poor escape from the trap. If an improvement in credit availability is insufficient, the rich cannot escape from the trap and, thus, economic development never occurs. Furthermore, even when the rich accumulate their wealth, a further improvement in credit availability would be required to help the poor to accumulate their own wealth. Income inequality and wealth inequality decrease considerably with a sufficient level of credit availability.

For simplicity, the degree of credit availability was exogenously given in our model. In future work, we intend to consider the joint and endogenous evolution of the degree of credit availability and economic development.

Appendix

A. Proof of Lemma 2

The proof takes three steps.

(i) Curves C and D intersect in the following range of e_{rt-1} where $e^{**} < e_{rt-1} < e^{D}$.

Proof: Because curve C is strictly concave, its curve locates above the 45 degree line in the following rate of e_{rt-1}: $e^{**} < e_{rt-1} < e^{*}$. The inequality $e^{**} < e^{D} < e^{*}$ shown by lemma 1 implies that curve C locates above the 45 degree line at the point where $e_{rt-1} = e^{D}$. Curve D, on the other hand, locates exactly on the 45 degree line where $e_{rt-1} = e^{D}$. Therefore, curve C locates above curve D at the point where $e_{rt-1} = e^{D}$. ♢

(ii) Curves C and D intersect at the point where $e_{t-1} = \hat{e}$ regardless of g.

Proof: Let us suppose that $e_{rt-1} = \hat{e}$. Equation (7) then becomes:

$$e_{rt} = (1-\alpha)f(\hat{e}) - \alpha\theta.$$

Using (25), (22) is rewritten as:

$$\begin{aligned} e_{rt} &= \frac{\eta}{\eta+(1-\eta)g}(1-\alpha)f(\hat{e}) + \frac{(1-\eta)g}{\eta+(1-\eta)g}[(1-\alpha)f(\hat{e}) - \alpha\theta] - \frac{\eta\alpha\theta}{\eta+(1-\eta)g} \\ &= (1-\alpha)f(\hat{e}) - \alpha\theta. \end{aligned}$$

Therefore, the two curves intersect at the point where $e_{rt-1} = \hat{e}$. ♢

(iii) Curves C and D can intersect only once.

Proof: The slope of curve C is given by $(1-\alpha)f'(e_{rt-1})$, while that of curve D is given by:

$$(1-\alpha)f'(e_{rt-1}) + (1-\alpha)\frac{(1-\eta)g}{\eta+(1-\eta)g}f''(e_{rt-1})e_{rt-1}.$$

Curve C is steeper than curve D at any investment level because of $f''(e_{rt-1}) < 0$. The two curves then can intersect only once. ♢

From the above (i), (ii), and (iii), we now reach the result $e^{**} < \hat{e} < e^{D}$.

B. A Perfect Credit Market

We consider a perfect credit market both in regimes I and II. We show that given the initial condition, $e^{DD} < e_{-1} = \eta b_{r,-1}$, a perfect credit market always helps the poor escape from a poverty trap.

Figure A1 shows the phase diagram which represents the dynamics of investment and wealth levels of the poor. We first see the curve $Z(e_{t-1})$ that represents $b_{pt} = 0$:

$$b_{pt} = (1-\alpha)[f(e_{t-1}) + f'(e_{t-1})(b_{pt-1} - e_{t-1})] - \alpha\theta = 0.$$

This equation is rewritten as

$$b_{pt-1} = -\frac{(1-\alpha)[f(e_{t-1}) - f'(e_{t-1})e_{t-1}] - \alpha\theta}{(1-\alpha)f'(e_{t-1})}. \tag{A1}$$

Differentiating (A1) with respect to e_{t-1}, we obtain

$$\frac{\partial b_{pt-1}}{\partial e_{t-1}}|_{b_{pt}=0} = \frac{(1-\alpha)f''(e_{t-1})[(1-\alpha)f(e_{t-1}) - \alpha\theta]}{[(1-\alpha)f'(e_{t-1})]^2}. \tag{A2}$$

The numerator in (A2) takes a negative value in the case that curve C lies above the horizontal axis. Therefore, the slope of curve $Z(e_{t-1})$ is negative. In addition, the curve $Z(e_{t-1})$ takes a value of zero at $\hat{e}$. Regime II is represented by the area above the curve $Z(e_{t-1})$.

Next, we consider the curve $G(e_{t-1})$ that represents $b_{pt} - b_{pt-1} = 0$:

$$b_{pt} - b_{pt-1} = (1-\alpha)[f(e_{t-1}) + f'(e_{t-1})(b_{pt-1} - e_{t-1})] - \alpha\theta - b_{pt-1} = 0.$$

This equation is rewritten as

$$b_{pt-1} = \frac{(1-\alpha)[f(e_{t-1}) - f'(e_{t-1})e_{t-1}] - \alpha\theta}{1-(1-\alpha)f'(e_{t-1})}. \tag{A3}$$

Differentiating (A3) with respect to e_{t-1}, we obtain

$$\frac{\partial b_{pt-1}}{\partial e_{t-1}}|_{b_{pt}-b_{pt-1}=0} = \frac{(1-\alpha)f''(e_{t-1})[(1-\alpha)(f(e_{t-1}) - \alpha\theta) - e_{t-1}]}{[1-(1-\alpha)f'(e_{t-1})]^2}. \tag{A4}$$

When e_{t-1} is lower than e^{**}, the numerator in (A4) takes a positive value because the curve C locates below the 45 degree line and $f''(e_{t-1}) < 0$. Where $e^{**} < e_{t-1} < \hat{e}$, on the other hand, this numerator becomes negative because the curve C locates above the 45 degree line. Therefore, while the slope of curve $G(e_{t-1})$ is positive in the range that $e_{t-1} < e^{**}$, it turns to be negative in the range that $e^{**} < e_{t-1} < \hat{e}$. The curve $G(e_{t-1})$ takes a value of zero at $\hat{e}$ because we have $b_{pt-1} = 0$, i.e., $b_{pt} - b_{pt-1} = 0$ at this point. (A1) and (A3) imply that the curve $G(e_{t-1})$ must lie above the curve $Z(e_{t-1})$ as long as e_{t-1} is lower than $\hat{e}$. If $b_{pt-1} > (<) G(e_{t-1})$, then b_{pt-1} increases (decreases).

The wealth level of the poor does not start to increase until the invest level exceeds $\hat{e}$. The curve $G(e_{t-1})$ takes a large positive value in the case that e_{t-1} locates in the neighborhood of the line N because the value of the denominator in (A3) is close to zero. The slope of $G(e_{t-1})$ is negative where $e_{t-1} < e^{*}$. It becomes positive where $e_{t-1} > e^{*}$ because the curve C locates below the 45 degree line. If $b_{pt-1} > (<) G(e_{t-1})$, then b_{pt-1} decreases (increases). On the other hand, Figure 2 implies the dynamcis of investment.

As shown in Figure A1, the wealth level of the poor starts to increase sooner or later in which the initial point is given by E because the investment level necessarily exceeds $\hat{e}$. The economy can eventually attain both income equality and wealth equality.[8]

[8] Galor and Tsiddon (1997) derived the Kuznets (Kuznets, 1955) curve in the model of a perfect credit market by assuming local home environment externality and global technological externality.

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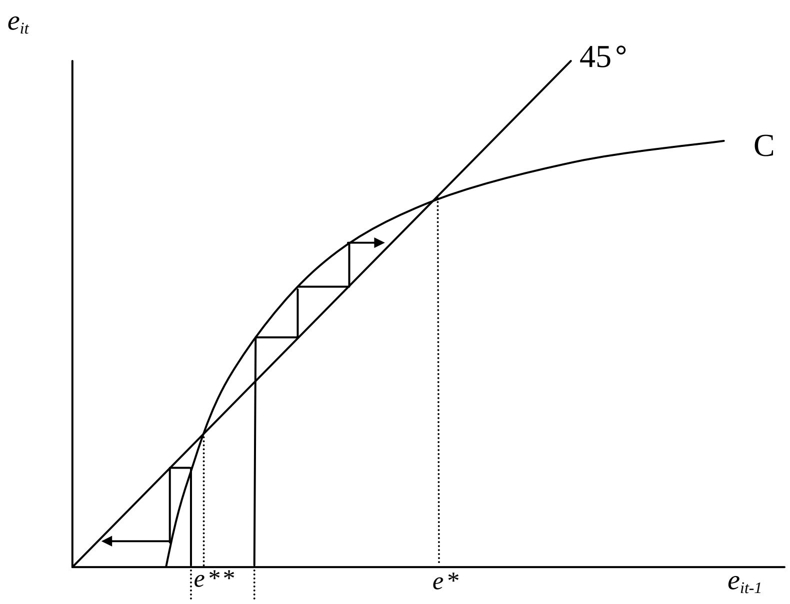

Figure 1. Dynamics of the investment level with no credit market

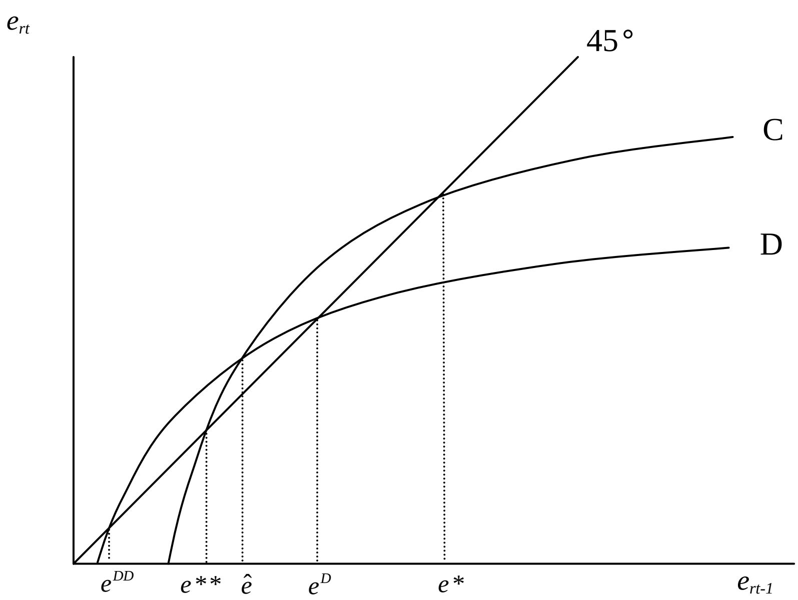

Figure 2. Dynamics of the investment level with a credit market

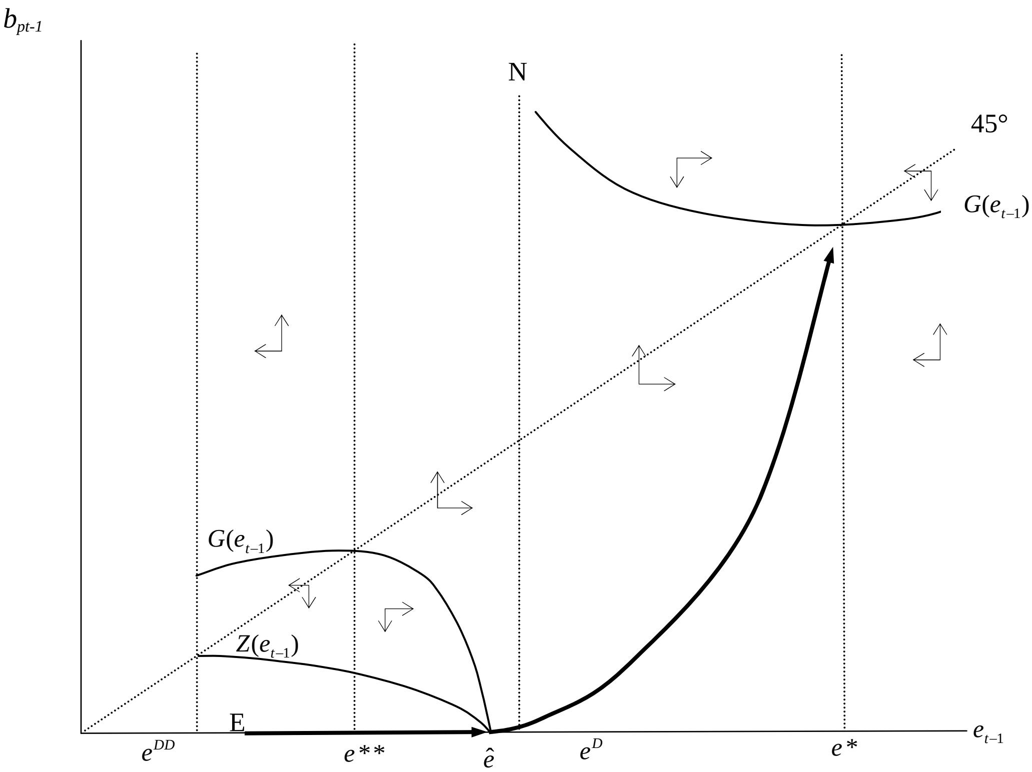

Figure A1. Phase diagram