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Fertility and Income Distribution with Increasing Costs of Education

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Discussion Paper No. 18

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Fertility and Income Distribution with Increasing Costs of Education*

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Abstract: Considering the price of education, we examine the dynamic relationship between education, fertility, and income distribution. We show that, although the number of children decreases with an increase in income, the ratio of total educational expenditure to income increases. We also show that, even if the education level of the poor increases and their number of children decreases, income inequality may start to rise. The demand for education by the rich makes the price of education too high for the poor. We also examine related policies. The child-benefit increases the possibility of inequality widening. Moreover, it decreases GDP per capita.

Keywords: Price of education, Fertility, Income inequality, Child-benefit, Scholarships.

JEL Classification: I20, J13, O15.

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1. Introduction

Income inequality in Japan has been increasing since the period of rapid economic growth, and a large increase in inequality has been recently identified.[1] The increasing inequality in income has caused controversy. As shown in Galor and Zeira (1993), given credit market imperfections, educational investment could become a key source of increasing inequality because it could strengthen the relationship regarding educational attainments between parents and their children. This was also pointed out in Teruyama and Ito (1994).

The cost of education has increased significantly because of large increases in tuition fees for private and public universities and the cost of supplementary private education. Figure 1 shows the ratio of educational expenditure to family income.[2] It has been increasing since the period of rapid economic growth. As argued in Kobayashi (2009), the large increase in private education cost has made educational expenditure a greater burden on household budgets. Figure 2, on the other hand, shows the fertility rate which is the rate of live birth per one hundred Japanese females. It has been decreasing since the period of rapid economic growth. The government has been trying to stop the decrease in the fertility rate by using policies such as the child-benefit.

How does the number of children relate to educational expenditure? How do the education levels and numbers of children of rich and poor evolve when increasing educational costs make income inequality between them widen? We examine

[1] See, for example, Tachibanaki (2005).

[2] All data used in Figures 1 and 2 were provided by the Statistical Bureau and the Director General for Policy Planning.

the dynamics of education and fertility of the rich and poor. We derive the price of education taking the increasing costs of education into account.[3] The price of education increases as the demand for education increases. Furthermore, following Moav (2002) and Galor and Moav (2004, 2006), we assume a non-homothetic utility function which allows zero expenditure for education. Educational expenditure is a convex function because of this assumption. Parents increase educational expenditure for their children when their incomes increase. Their number of children, on the other hand, decreases. That is, the quality of children is preceded more than the quantity with an increase in income. However, not only the total educational expenditure but also its ratio to income increase. That is, the educational expenditure becomes a greater burden on household budgets as the income level increases.

Assuming credit market imperfection, we examine income inequality. When the basis of human capital is low, even if the initial gap in education levels between the rich and the poor is small, the poor cannot afford education in the long run.[4] This is because the demand for education by the rich makes the price of education too high. Furthermore, when the education levels of both rich and poor increase, their numbers of children decrease. That is, income inequality increases, even though their income levels increase and the fertility rate of the economy decreases. The

[3]Using a model in which outputs depend partially on customers as inputs, Rothschild and White (1995) showed that prices that charge customers for what they get on net from the firm are competitive and support efficient allocation. This price is essentially the same as the price of education in our model.

[4]If elementary education forms its basis, the degradation of elementary education would increase income inequality. See Nakajima and Nakamura (2009b). Tabata (2003) showed that a decrease in public expenditure on basic education would make an economy fall into a poverty trap.

education level of the poor starts to decrease when the price of education increases more rapidly than their incomes. The rising inequality then gradually becomes serious. While the number of children of the rich continues decrease, the number of children of the poor turns to increase. Thus, changes in the fertility rate of the economy depend on the ratio of the rich to the total population.[5]

We must emphasize the following points. Galor and Weil (2000), Galor and Moav (2002), de la Croix and Doepke (2003), Tabata (2003), and Yakita (2008) among others showed that, while population growth is high in the early stages of economic development, fertility decreases in the later stages.[6] We consider developed economies in which the cost of education increases significantly. We show that, even when the number of children decreases, the ratio of the total amount of educational expenditure to income increases. Galor and Moav (2004, 2006) presented dynamic models that explain both inequality and the process of economic development. In their models, income inequality decreases when the poor start to invest in their human capital. By considering the price of education in a model that considers fertility, we complement Galor and Moav (2004, 2006). We show that, even if the education level of the poor exceeds the poverty trap threshold, they may not be able to receive education in the long run because the demand for education by the rich makes the price of education too high for them. Income inequality may start to increase even when the numbers of children of rich and poor both decrease.

[5] Assuming individuals of the quality type and quantity type, Galor and Moav (2002) examined the dynamics of those ratios and the rate of technological progress. Considering asymmetric behavior with respect to fertility and education between individuals stemming from income inequality, Momota (2009) showed that economic growth may slow down when a fertility transition starts.

[6] See Galor (2005) that carried out an extensive survey.

Nakajima and Nakamura (2009a,b) also considered the effect of the increasing costs of education on income inequality. However, they did not consider fertility. In addition, we examine the effects of related policies, including the child-benefit and scholarships.[7] The benefit makes parents prefer the number of children more than their quality. That is, compared with no child-benefit, the numbers of children of rich and poor both increase. However, it worsens the condition of inequality widening. It also decreases GDP per capita. Scholarships, on the other hand, increase the education level. It can decrease the inequality. Compared with no scholarships, GDP per capita increases and the numbers of children of rich and poor both decrease. However, when the amount of scholarships is sufficiently large, an increase in scholarships does necessarily decrease the fertility rate because the scholarship alleviates the burden of educational expenditure on household budgets.

The rest of the paper is organized as follows. Section 2 explains our model and Section 3 describes the dynamic relationship among education, fertility, and income distribution. We also examine policies that include the child-benefit and scholarships. We conclude in Section 4 with a brief summary and a few remarks.

2. The model

We consider a closed overlapping-generations economy. If parents decide to spend on education, their children receive this education in the first period. Individuals work in the second period. They decide their consumption level, the number of children,

[7]Glomm and Ravikumar (1992), Fernandez and Rogerson (1996), Zhang (1996), Bräuninger and Vidal (2000), and Bénabou (1996, 2000) examined the effects on income inequality of educational and fiscal policies, such as public versus private education, public education provided at the community level, socioeconomic stratification, wealth redistribution, and education finance.

and the education level of their children. The population born in period t is L_t. The numbers of rich and poor are represented as L_{rt} and L_{pt}, respectively. That is, we have $L_t = L_{rt} + L_{pt}$. Their initial education levels are denoted as $e_{r,-1}$ and $e_{p,-1}$, respectively. We assume that $e_{r,-1} > e_{p,-1}$. We represent the ratio of the rich to the total population as $\lambda_t \equiv L_{rt}/L_t$. We consider a consumption goods sector and an educational sector. Firms in the consumption goods sector are perfectly competitive. A nonprofit organization runs the educational institution.

2.1 Individuals

We first describe how education forms the stock of human capital. For simplicity, the capital stock of an individual is assumed to be linear:

$$h(e_{it-1}) = \delta + e_{it-1}, \tag{1}$$

where $i = r, p$. $\delta > 0$. The human capital stocks of the rich and poor are $h(e_{rt-1})$ and $h(e_{pt-1})$, respectively, where e_{rt-1} and e_{pt-1} are, respectively, the levels of education of the rich and poor, which are received in period $t-1$.

While the rich and poor have the same constant term, we represent the level of higher education by e_{it-1}. This positive term ensures that individuals with no higher education can live. In contrast, they can receive different levels of higher education because, given the borrowing constraints, educational expenditure depends on the incomes of parents.

Individuals obtain their incomes by supplying their labor. They care about the consumption level, the number of children, and the bequest level for their children. We consider the cost of child raising as the opportunity cost. While we assume credit market imperfection, for simplicity, loans are assumed to be unavailable. Therefore, consumption and bequests for their children are paid for out of income.

The utility maximization problem of an individual born in period $t-1$ is written as:

$$\max_{c_{it},n_{it},e_{it}} \beta_1 \ln c_{it} + \beta_2 \ln n_{it} + (1-\beta_1-\beta_2)\ln(b_{it}+\theta), \tag{2}$$

$$s.t. \qquad (1-\eta n_{it})I_{it} = c_{it} + n_{it}b_{it}, \tag{3}$$

where $i = r, p$. We assume that $0 < \beta_1, \beta_2, \eta < 1$, $0 < 1-\beta_1-\beta_2 < 1$, and $1-\eta n_{it} < 1$. n_{rt} and n_{pt} are the numbers of children of the rich and poor, respectively. I_{rt} and I_{pt} are the incomes per unit of labor of the rich and poor, respectively. η is the time of child raising per a child. c_{rt} and c_{pt} are the consumption levels of the rich and poor, respectively. b_{rt} and b_{pt} are the bequest levels per child of the rich and poor, respectively.

The bequest level in the utility function represents the altruistic bequest motive, i.e., the 'joy of giving'. Parameter θ allows a zero bequest.[8] We assume that $\beta_2 > (1-\beta_1-\beta_2)$, which is required to assure the existence of equilibrium. This assumption implies that parents care about the number of children more than expenditure for bequests because expenditure with no children does not make sense.

For simplicity, the bequest is used only for educational expenditure, i.e., it cannot be used for consumption:[9]

$$b_{it} = p_t e_{it}, \tag{4}$$

[8] If we assume that parental preferences depend on the human capital stock of their children, given the level of parental income, an increase in the price of education decreases educational expenditure per a child. Furthermore, the dynamics of the education level would be complicated. See Nakajima and Nakamura (2009b).

[9] We consider that c_t includes consumption of parents and their children.

where p_t is the price of education.

The first-order conditions of the utility maximization problem imply that educational expenditure is a convex function because of θ. When the following condition holds:

$$(1-\beta_1-\beta_2)\eta I_{it}-\beta_2\theta \leq 0,$$

there is no educational expenditure. We then have:

$$p_t e_{it}=0, \tag{5}$$

$$n_{it}=\frac{\beta_2}{(\beta_1+\beta_2)\eta}, \tag{6}$$

$$c_{it}=(1-\eta n_{it})I_{it}=\frac{\beta_1}{\beta_1+\beta_2}I_{it}. \tag{7}$$

When the income level is low, educational investment does not occur. We compare the ratio of the marginal benefit of an additional child to the marginal cost with the ratio of the marginal benefit of an additional bequest to the marginal cost at $b_t=0$. The former is then greater than the latter because of a low income level and θ. The number of children takes a constant value because the cost of child raising is proportionate to the income. We assume that $\beta_2/[(\beta_1+\beta_2)\eta]>1$, which implies $n_{it}>1$. If this inequality does not hold, the number of total population always decreases regardless of changes in income distribution. The ratio of consumption to income is constant.

Next, when the income level is high enough to satisfy:

$$(1-\beta_1-\beta_2)\eta I_{it}-\beta_2\theta > 0,$$

educational expenditure is positive. The first-order conditions are:

$$p_t e_{it}=\frac{(1-\beta_1-\beta_2)\eta I_{it}-\beta_2\theta}{\beta_2-(1-\beta_1-\beta_2)}, \tag{8}$$

$$n_{it} = \frac{\beta_2 - (1 - \beta_1 - \beta_2)}{\beta_1 + \beta_2} \frac{1}{\eta - \theta / I_{it}}, \tag{9}$$

$$c_{it} = \frac{\beta_1}{\beta_1 + \beta_2} I_{it}. \tag{10}$$

When the income level exceeds the threshold which is represented by $\beta_2\theta/[(1 - \beta - \beta_2)\eta]$, parents start to give their children the bequest, i.e., educational investment starts.[10] The educational expenditure increases as the income level increases.

Using (9), we have:

$$\frac{\partial n_{it}}{\partial I_{it}} < 0 \quad and \quad \frac{\partial^2 n_{it}}{\partial I_{it}^2} > 0.$$

(9) is equal to (6) when $(1 - \beta_1 - \beta_2)\eta I_{it} = \beta_2\theta$. The number of children decreases as the income level increases. However, its decrease becomes small with an increase in income. Figure 3 shows the relationship between the number of children and the income level. An increase in income increases the opportunity cost of child raising. It induces parents to spend educational expenditure for their children and reduce their number.

Furthermore, using (8) and (9), the ratio of total educational expenditure to income can be written as:

$$\frac{n_{it} p_t e_{it}}{(1 - \eta n_{it}) I_{it}} = \frac{(1 - \beta_1 - \beta_2)\eta I_{it} - \beta_2\theta}{[\beta_1 + (1 - \beta_1 - \beta_2)]\eta I_{it} - (\beta_1 + \beta_2)\theta}.$$

By considering the opportunity cost of child raising, we represent the income level as $(1 - \eta n_{it}) I_{it}$. This implies that:

$$\partial \frac{n_{it} p_t e_{it}}{(1 - \eta n_{it}) I_{it}} / \partial I_{it} > 0.$$

[10]Considering fertility and the timing of educational investment, Yakita (2008) showed the endogenous shift from an exogenous growth phase to an endogenous growth phase.

When the income level increases, the ratio of total educational expenditure to income increases. That is, although the number of children decreases with an increase in income, the burden of educational expenditure on household budgets increases. This is because of the more-than-offsetting increase in the educational expenditure per a child.

The relationship between consumption and income per labor unit does not change regardless of the income level. However, the ratio of consumption to income which is represented by $c_{it}/[(1-\eta n_{it})I_{it}]$ decreases with an increase in income.

2.2 Educational sector and consumption goods sector

We first consider the institution of education. We assume that teachers are among the rich because their education level is higher than that of the poor. Education is an outcome of collaboration between teachers and students. The total amount of education is assumed to be subject to a Cobb–Douglas production function:

$$e_{at}L_t^S = (h(e_{rt-1})L_t^T)^{\alpha}(L_t^S)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{11}$$

where e_{at} is the average educational level per student received in period t, and L_t^T and L_t^S are the numbers of teachers and students in period t, respectively.

The price of education is defined as p_t. The institution has a balanced budget:

$$p_t e_{at} L_t^S = h(e_{rt-1})L_t^T, \tag{12}$$

where the left side represents the total amount of tuition, and the right side represents the total wage cost of teachers.

The price of education is determined by a zero profit condition. Using (11) and (12), the price of education can be derived as:

$$p_t = e_{at}^{(1-\alpha)/\alpha}. \tag{13}$$

The price of education increases with an increase in the average education level because of diminishing returns to teachers.

Next, we describe the consumption goods sector in which there are many competitive firms. While the rich are employed in the education and consumption goods sectors, the poor are employed in only the latter. For simplicity, the production function is assumed to be linear:

$$Y_t = h(e_{pt-1})(1 - \eta n_{pt})L_{pt-1} + h(e_{rt-1})(1 - \eta n_{rt})(L_{rt-1} - L_t^T). \tag{14}$$

Normalizing the price of consumption goods to unity, the income level is equal to the level of human capital stock, i.e., we have $I_{it} = h(e_{it-1})$.

3. Education, fertility, and income distribution

3.1 Dynamics

We first see the price of education. The price of education depends on the average education level:

$$p_t = [\lambda_t e_{rt} + (1 - \lambda_t)e_{pt}]^{(1-\alpha)/\alpha},$$

where $\lambda_t \equiv L_{rt}/L_t$ which represents the ratio of the rich to the total population.

Furthermore, using (8) and (13), the price of education can be represented as the weighted average of demand for education by the rich and poor:

$$p_t = [A(\lambda_t e_{rt-1} + (1 - \lambda_t)e_{pt-1}) + B]^{1-\alpha}, \tag{15}$$

where

$$A \equiv \frac{(1 - \beta_1 - \beta_2)\eta}{\beta_2 - (1 - \beta_1 - \beta_2)} \quad and \quad B \equiv \frac{(1 - \beta_1 - \beta_2)\eta\delta - \beta_2\theta}{\beta_2 - (1 - \beta_1 - \beta_2)}.$$

Using (8) and (15), the dynamics of education for the rich and poor, respectively, are:

$$e_{rt} = \frac{Ae_{rt-1} + B}{p_t} = \frac{Ae_{rt-1} + B}{[A(\lambda_t e_{rt-1} + (1 - \lambda_t)e_{pt-1}) + B]^{1-\alpha}}, \tag{16}$$

$$e_{pt} = \frac{Ae_{pt-1} + B}{p_t} = \frac{Ae_{pt-1} + B}{[A(\lambda_t e_{rt-1} + (1 - \lambda_t)e_{pt-1}) + B]^{1-\alpha}}. \tag{17}$$

The dynamics of education for the rich and poor are mutually dependent through the price of education because the price of education is the weighted average of the demands by the rich and the poor. Using (9), the dynamics of λ_t are:

$$\lambda_t = \frac{n(e_{rt-1})\lambda_{t-1}}{n(e_{rt-1})\lambda_{t-1} + n(e_{pt-1})(1 - \lambda_{t-1})}, \tag{18}$$

where $n_{it} = n(e_{it-1})$, $i = r, p$.

The dynamics of our model are described by (16), (17), and (18). When we linealize this system around steady states, the dynamics of education for the rich and poor do not depend on the ratio of rich because of no difference in their numbers of children between the rich and the poor on the steady states.

The dynamics of education for the rich and poor are crucially influenced by the sign of B. The sign of B depends on the basis of the human capital stock, δ. A low δ implies a negative B. Assuming $B < 0$, we discuss the dynamics. Figure 4 shows the phase diagram. Given the ratio of the rich, this figure shows the conditional dynamics of education levels of rich and poor.[11] Changes in the ratio of the rich do not affect the dynamics qualitatively. The steady state, O, that implies a poverty

[11] Galor and Weil (2000) showed the conditional dynamics of education and effective resources per worker which were conditioned by the size of population. Galor and Moav (2002) also showed the conditional dynamics of technological progress and effective resources which were conditioned by the ratio of individuals of the quality type.

trap, is stable. The steady state, C, which is the threshold of the poverty trap, is stable. The steady state, D, is a saddle point. The saddle path always exists on the 45 degree line because of the homogeneity between the rich and the poor except for their initial education levels. Furthermore, the education levels on the steady states, which are represented by e^* and e^{**} are not affected by the ratio of the rich. However, the lines, $\Delta e_{rt} = 0$ and Δe_{pt} move with a change in its ratio.

Consider the initial point, E. This indicates that the initial education level of the poor is higher than the poverty trap threshold. Furthermore, the initial gap in education levels between the rich and the poor is small. The education levels of the rich and poor temporarily increase. The price of education is not too high for the poor to spend on education because of the small difference in the income levels between the rich and the poor. However, the education level of the rich increases more than that of the poor because the price of education is low for the rich. Thus, the inequality in income between them increases. The numbers of children of the rich and poor are both decreasing. Because the price of education increases more than the incomes of the poor, the education level of the poor starts to decrease sooner or later. The number of children of the poor now increases. While the education level of the rich continues to increase, their number of children continues to decrease. Changes in the number of total population depend on the numbers of children of the rich and poor and the ratio of the rich.

The education level of the poor eventually becomes zero because the demand for education by the rich makes the price of education too high for the poor.[12] The

[12]The price of education may not continue to increase because the ratio of the rich decreases. However, its price is still high for the poor.

number of children of the poor becomes constant. The dynamics of education of the rich are now the same as those of homogenous individuals:

$$e_{rt} = (Ae_{rt-1} + B)^{\alpha}. \tag{19}$$

Their education level converges to e^*. The number of children of the rich is also constant in the long run. Because the number of children of the poor is always larger than that of the rich, the ratio of the rich asymptotically approaches zero regardless of the initial value. The number of total population increases in the long run because of the assumption that the number of children written in (6) is larger than unity.

The following proposition is made for the case of $B < 0$.

Proposition 1: *If $B < 0$ holds, the poor cannot afford education in the long run regardless of their initial education level. Even when the education level of the poor increases and their number of children decreases, income inequality between the rich and the poor widens. Because the demand for education by the rich makes the price of education high, the education level of the poor turns to decrease and their number of children turns to increase.*

3.2 Effects of policies

Are there effective policies to promote income equality and macroeconomic development? We consider consumption taxes used for the child-benefit and scholarships because those taxes do not affect educational expenditure.

We first examine the child-benefit. The budget constraints on an individual written in (3) can be rewritten as:

$$(1 - \eta n_{it})I_{it} + n_{it}a = (1 + d_t)c_{it} + n_{it}b_{it}, \tag{20}$$

where $i = r, p$. $a > 0$ which represents the child-benefit, and $d_t c_{it}$ represents the amount of consumption taxes.

While the tax revenue is used for the child-benefit, we consider a balanced budget of the government. For simplicity, the child-benefit per a child is assumed to be given equally among parents:

$$[\lambda_{t-1} n_{rt} + (1 - \lambda_{t-1}) n_{pt}] a = [\lambda_{t-1} c_{rt} + (1 - \lambda_{t-1}) c_{pt}] d_t.$$

Given a, d_t is set to satisfy this equation. This benefit implies that the poor benefit more than the rich because the number of children of the poor is larger than that of the rich. Furthermore, the amount of consumption taxes levied on the poor is smaller than that on the rich.

The educational expenditure is written as:

$$p_t e_{it} = A e_{it} + B_a,$$

where $i = r, p$, and

$$B_a \equiv \frac{(1 - \beta_1 - \beta_2)(\eta\delta - a) - \beta_2 \theta}{\beta_2 - (1 - \beta_1 - \beta_2)}.$$

The child-benefit decreases educational expenditure because it decreases B_a. In addition, its benefit increases the possibility of income inequality widening because of the inequality, $B > B_a$.

We consider steady states for the case in which $B_a < 0$ holds. The rich receive education as long as their initial education level exceeds the poverty trap threshold.[13] Their education level at the high-level steady state is represented as:

$$e_r = (A e_r + B_a)^{\alpha}. \tag{21}$$

[13]Even if their initial education level does not exceed e^{**}, they could afford education because the demand for education by the poor makes the price of education low for the rich.

Note that $e_r = e^*$. The child-benefit decreases the education level of the rich because it makes parents care about the number of children more than their quality. Their number of children is written as:

$$n_r = \frac{\beta_2 - (1 - \beta_1 - \beta_2)}{\beta_1 + \beta_2} \frac{1}{\eta - (a + \theta)/(\delta + e_r)}. \quad (22)$$

The number of children increases because of the child-benefit. However, the effect on their consumption level is ambiguous and, thereby, the effect on their utility level is also ambiguous.

The poor cannot afford education in the long run regardless of their initial education level. That is, we have $e_p = 0$. Their number of children is represented as:

$$n_p = \frac{\beta_2}{\beta_1 + \beta_2} \frac{1}{\eta - a/\delta}. \quad (23)$$

The child-benefit increases the number of children, although they cannot receive education. Because the effect on their consumption level is ambiguous, their utility level might not increase. Therefore, compared to the case of no child-benefit, the benefit always decreases GDP per capita although it increases the fertility rate of the economy.[14] Furthermore, the utility levels of the rich and poor might not increase.

Next, we consider scholarships. The budget constraints of households are rewritten as:

$$(1 - \eta n_{it}) I_{it} = (1 + d_t) c_{it} + n_{it} b_{it}, \quad (24)$$

where $i = r, p$.

The tax revenue is used for scholarships. For simplicity, the scholarships are

[14] If we considered the child-benefit only for the poor, this conclusion would be strengthened.

assumed to be granted equally among students:

$$[\lambda_{t-1}n_{rt} + (1-\lambda_{t-1})n_{pt}]s = [\lambda_{t-1}c_{rt} + (1-\lambda_{t-1})c_{pt}]d_t,$$

where $s > 0$ which represents the scholarship amount.

Given s, d_t is set to satisfy this equation. A constant s implies that the poor benefit more than the rich because the number of children of the poor is larger than that of the rich. Moreover, the amount of consumption taxes levied on the poor is smaller than that on the rich.

The educational expenditure is written as:

$$p_t e_{it} = Ae_{it} + B_s,$$

where $i = r, p$, and

$$B_s \equiv \frac{(1-\beta_1-\beta_2)\eta\delta - \beta_2\theta}{\beta_2 - (1-\beta_1-\beta_2)} + s.$$

The scholarship increases educational expenditure because of an increase in B_s. We have $B_s > B$. A sufficient amount of scholarships can make an economy attain income equality because it changes from one regime having a saddle point to two regimes having a stable steady state. This implies that parents always spend on education for their children regardless of their education levels.

Let us consider a steady state for the case in which $B_s > 0$ holds. The rich and poor can attain the same education level:

$$e^* = (Ae^* + B_s)^\alpha. \tag{25}$$

The scholarship increases the education level. The number of children, on the other hand, becomes:

$$n^* = \frac{\beta_2 - (1-\beta_1-\beta_2)}{\beta_1+\beta_2}[\eta - \frac{\theta - (\beta_2 - (1-\beta_1-\beta_2))s/\beta_2}{\delta + e^*}]^{-1}. \tag{26}$$

That is, we have $n^* \equiv n_r = n_p$. Compared with no scholarships, the number of children always decreases. However, when the amount of scholarships is sufficiently large, the sign of $\partial n^*/\partial s$ can become positive. An increase in scholarships has two effects. The first is to the relationship between s and n^* in (26). An increase in s decreases n^* because it makes parents prefer the quality of children than their quantity. The second is to the relationship between e^* and n^* in (26). The scholarship increases the education level, i.e., the income level. If the amount of scholarships is enough to make $\theta - (\beta_2 - (1 - \beta_1 - \beta_2))s/\beta_2$ negative, which implies $B_s > 0$, an increase in e^* rises n^*. A large amount of scholarships helps individuals to attain a high education level. Because this alleviates the burden of its expenditure on household budgets, parents can afford to have more children. Therefore, an increase in scholarships does not necessarily decrease the number of children.

Figure 5 shows the phase diagram for which $B_s > 0$ holds. Given the ratio of the rich, this figure shows the conditional dynamics of education levels of rich and poor. Changes in the ratio of the rich do not affect the dynamics qualitatively. A high-level steady state, but not multiple steady states, exists. Compared with Figure 4, the slopes of $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ on e^* are opposite. Although the lines, $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ move with a change in the ratio of the rich, e^* is not affected by its ratio.

When the education levels of both the rich and poor increase, the price of education would increase. However, the income level of the poor is now sufficient for them to afford the increasing cost of education. Any initial value converges to e^*. Income inequality disappears in the long run. The numbers of children of rich and poor both, on the other hand, decrease with increases in their education levels.

While the ratio of the rich converges to a value, its value depends on the initial value and parameters. If $n^* < 1$ holds, the total population decreases. Compared with the case with $B < 0$, the education levels of both the rich and poor increase. Therefore, the GDP per capita increases. However, the fertility rate of the economy decreases. The income level of the poor increases and their educational expenditure also increases although their number of children decreases. If these positive effects are larger than the negative effect, their welfare level could increase because of the scholarship.[15]

The following proposition is made concerning the child-benefit and scholarships.

Proposition 2: *The child-benefit increases the possibility of inequality widening. Although it increases the fertility rate, it decreases GDP per capita. A sufficient amount of scholarships, on the other hand, can decrease income inequality. Although it decreases the fertility rate, it increases GDP per capita.*

4. Concluding remarks

We showed that, while the number of children decreases with an increase in income, the burden of educational expenditure on household budgets increases. The non-homothetic utility function which allows a zero educational expenditure implies that the quality of children is preceded more than the quantity. In addition, if the basis of human capital is small, income inequality increases even if the initial gap in education levels between the rich and the poor is small. When the education level of the poor increases, their number of children decreases. Income inequality starts to widen in these periods. The education level of the poor eventually decreases because

[15] Changes in the welfare level of the rich also depend on these effects.

a large demand for education by the rich makes the price of education too high. The number of children of the poor then increases and converges to a constant value. The number of children of the rich, on the other hand, decreases as their incomes increase.

We examined two polices which included the child-benefit and scholarships. The child-benefit increases the possibility of income inequality widening. Furthermore, it decreases GDP per capita because it decreases the education level of the rich and increases the numbers of children of the rich and the poor. Scholarships, on the other hand, can enhance income equality. Because the education levels of both the rich and poor increase, GDP per capita increases. However, the numbers of children of the rich and the poor decrease.

If the government tries to increase subsidies for both child raising and educational expenditure, neither the education level nor the fertility rate is likely to increase because the effects might be cancelled out. Therefore, the government should carefully use those policies to attain income equality and not to decrease the fertility rate.

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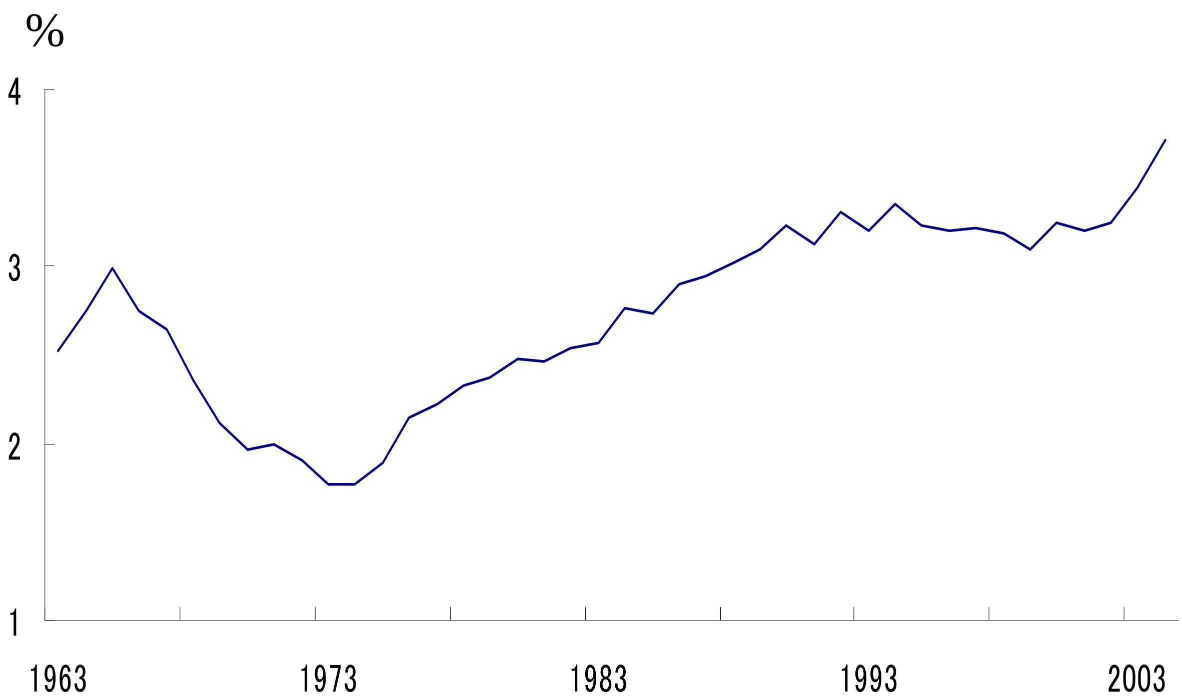

Figure 1. Ratio of educational expenditure to family income in Japan

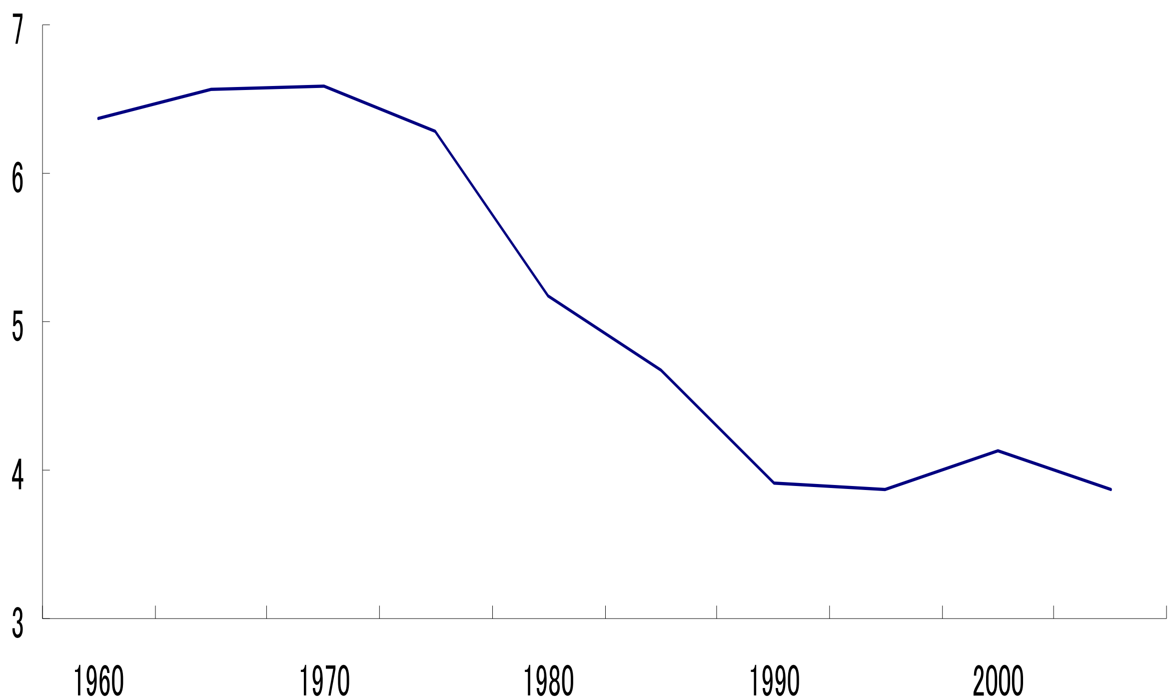

Figure 2. Fertility rate in Japan

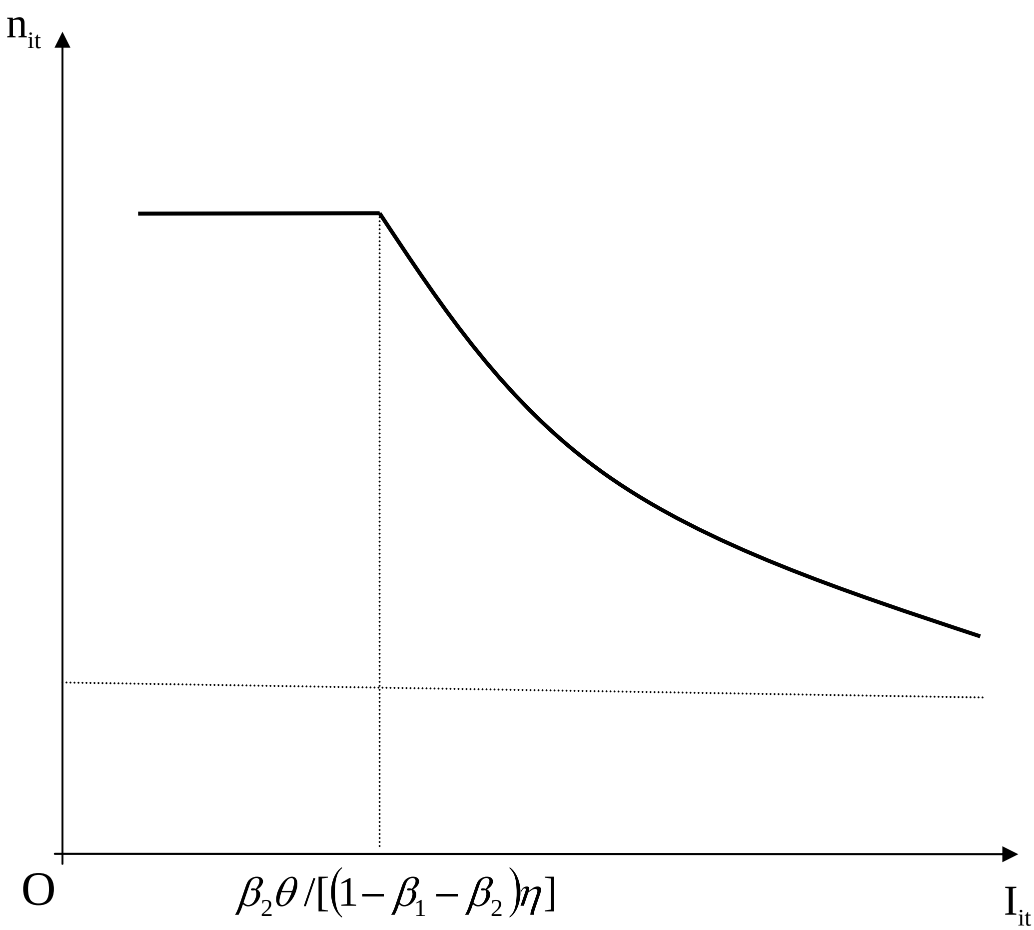

Figure 3. Relationship between the fertility rate and the income level

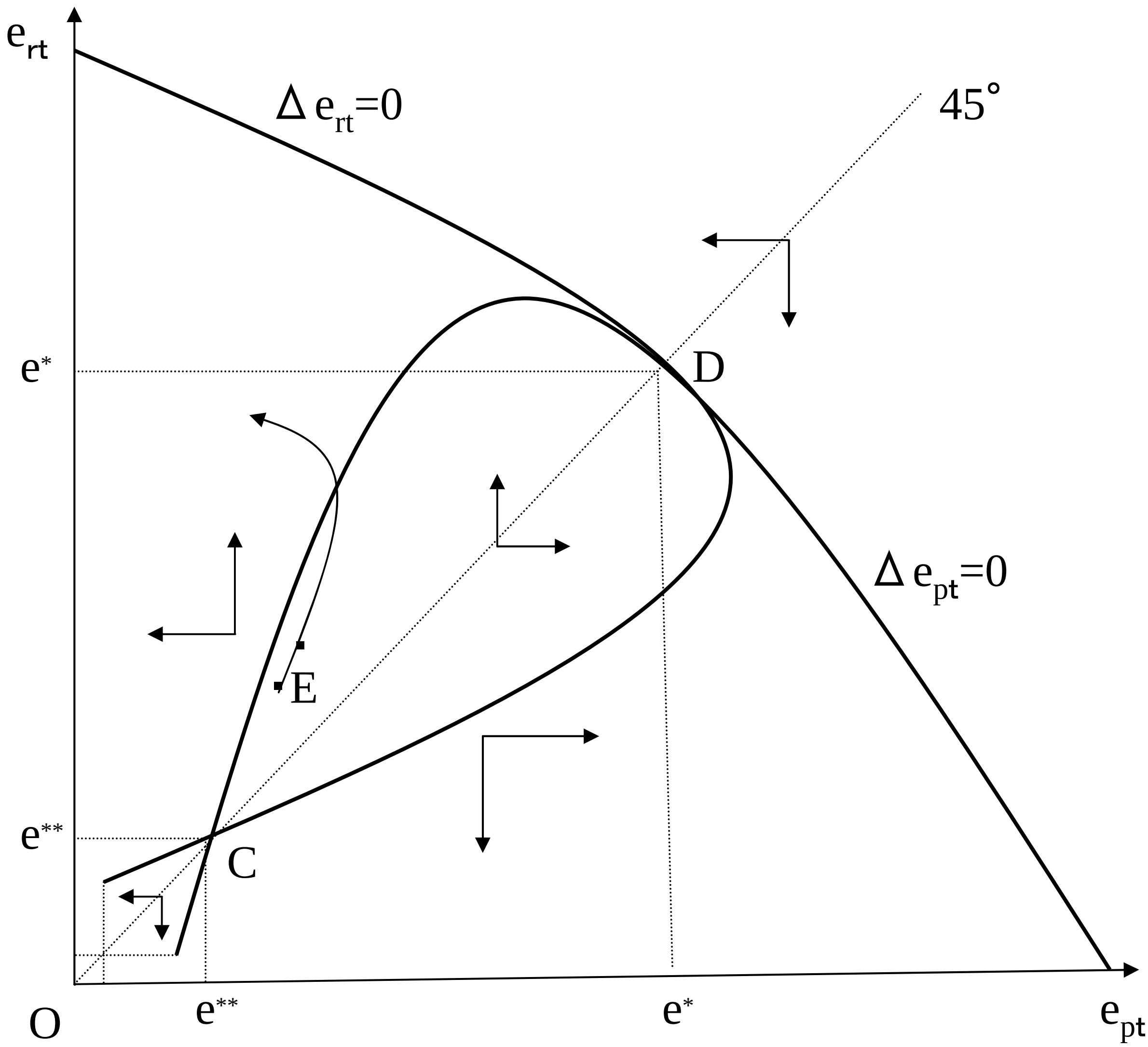

Figure 4. Conditional dynamics in the case of B<0

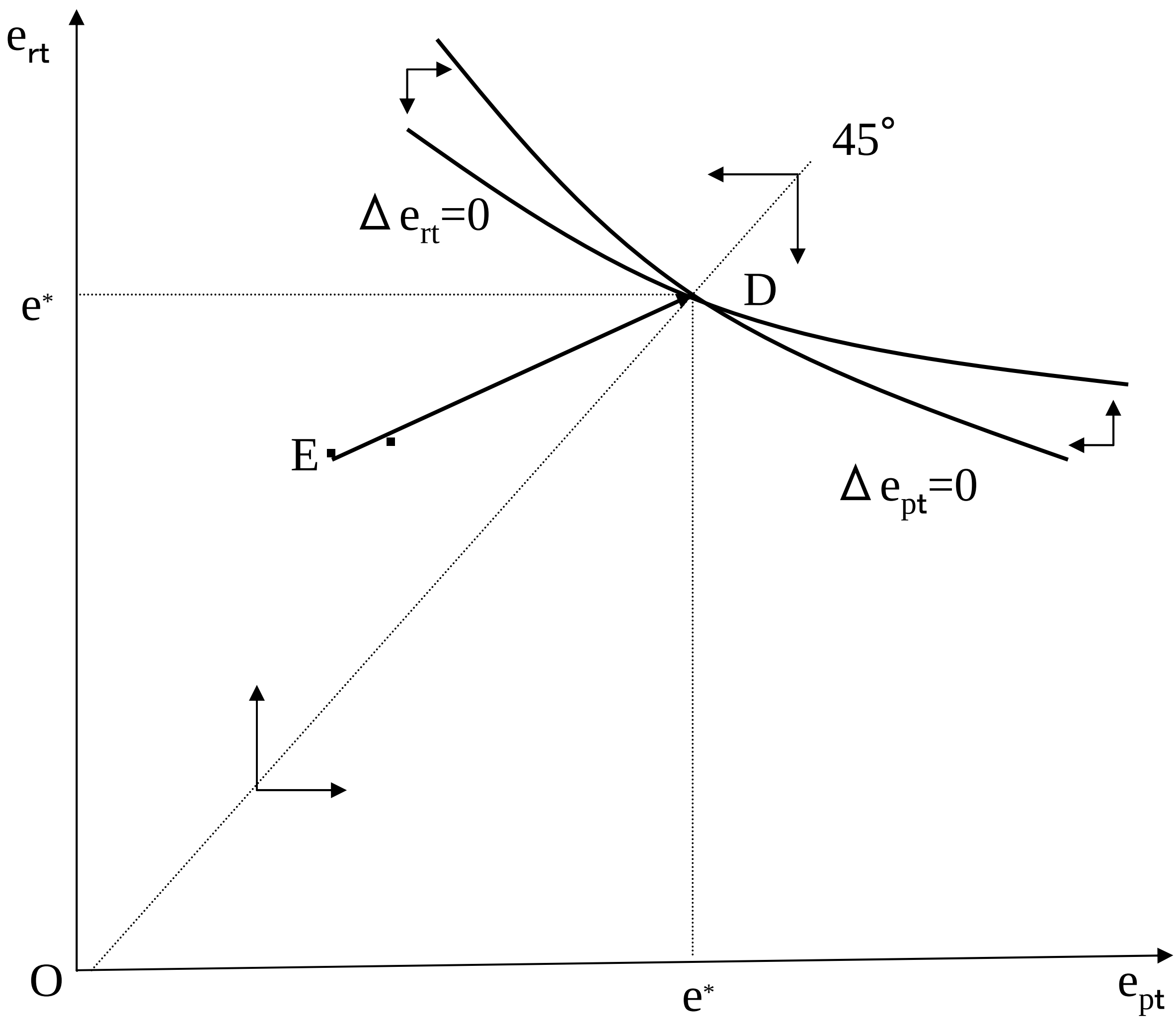

Figure 5. Conditional dynamics in the case of $B_s > 0$