CREI

Discussion Paper Series

Achievement Tests in Japan: An Econometric Analysis

Hideki Nakamura_
Faculty of Economics,
Osaka City University

March 30, 2009
Discussion Paper No. 14

Center for Research on Economic Inequality (CREI)
Graduate School of Economics
Osaka City University

3-3-138 Sugimoto, Sumiyoshi-ku,
Osaka 558-8585, Japan
http://www.econ.osaka-cu.ac.jp/CREI/index.html

CREI Discussion Paper Series

Achievement Tests in Japan: An Econometric Analysis

Hideki Nakamura_
Faculty of Economics,
Osaka City University

March 30, 2009
Discussion Paper No. 14

経済格差研究センター（CREI）は、大阪市立大学経済学研究科重点研究プロジェクト「経済格差と経済学－異端・都市下層・アジアの視点から－」（2006~2009 年）の推進のため、研究科内に設置された研究ユニットである。

Achievement Tests in Japan: An Econometric Analysis

Hideki Nakamura*

Faculty of Economics, Osaka City University

Abstract: By using prefectural data of achievement tests in Japan, we investigate empirically which factors affect the achievement of elementary school pupils and junior high school students. In our regressions, we find that the ratio of grandparents living with their grandchildren has a strong positive effect on achievement. Family income and the ratio of people having completed university, which is a proxy of the education level of parents, affect achievement positively. The quality of teachers, but not the quantity, is important for achievement. Public education expenditure does not contribute at all.

Keywords: achievement, home environment, quality of teachers;

JEL Classification: I20, I21, I28.

*Correspondence: Hideki Nakamura, Faculty of Economics, Osaka City University, 3-3-138 Sugimoto, Sumiyoshi, Osaka 558-8585

Phone:+81-6-6605-2271, Fax:+81-6-6605-3066, E-mail: hnakamur@econ.osaka-cu.ac.jp

1. Introduction

The Japanese government practiced less strenuous school education called *yutori kyouiku* from the second half of the 1990s. It has been implemented thoroughly since 2002. The reform of the education system aimed to reduce the possible negative effects of examinations, and the number of school days and course content were also reduced. A special class called *sougou gakushuu* was also adopted to develop moral and social skills. However, Kariya (2007) argued that, because this education reform did not address the state of the actual education system, it increased the gap between hard working students and other students.[1] Nishimura (2003) also argued that less strenuous school education reduced enthusiasm and competition among students. He pointed out a deterioration of public schools and an increase in the prosperity of supplementary private schools.

The government recently reviewed this less strenuous school education system. In 2007 and 2008, the Ministry of Education, Culture, Sports, Science and Technology conducted achievement tests to assess the scholastic ability of elementary school and junior high school students. Although the results of the achievement test have been widely examined, there has been little econometric analysis of the results. Thus, by using prefectural data in our regressions, we examine empirically which factors affect the achievement of elementary school pupils and junior high school students.

We consider both home and school environments as explanatory variables to explain achievement scores. We obtain similar results for elementary school pupils

[1] Kariya (2001) explained the appearance of an incentive gap between these students.

and junior high school students. The importance of family income is confirmed for both groups of students because it can affect supplementary private education. The importance of family income would be higher for students at junior high schools than for students at elementary schools. The proportion of households with members aged 65 years and over influence achievement positively and significantly because grandparents can watch and help their grandchildren. The proportion of people having completed university, as a proxy of parental education level, is positively significant. Moreover, we find weak evidence that the number of universities has an externality effect on achievement.

Because skilled teachers can decrease the ratio of long-term absentees from school, the ratio of long-term absentees from school is used as a measure of the quality of teachers. The quality of teachers is important for achievement as an indicator of school environment. However, the number of teachers per student, i.e., the quantity of teachers is not significant. Public education expenditure does not contribute to achievement at all. We also find a negative effect of the ratio of female teachers on junior high schools' achievement, but not on elementary schools' achievement. Because a high ratio of female teachers implies a high ratio of young teachers in junior high schools, the result points out the importance of the experience of teachers.

The rest of the paper is organized as follows. Section 2 examines what factors affect achievement. We estimate achievement of elementary school pupils in Section 3.1 and junior high school students in Section 3.2. We conclude Section 4 with a brief summary and a few remarks.

2. What factors affect achievement?

The prefectural data of the achievement of public schools in 2007 and 2008 were taken from the Japan Students Services Organization. Mathematics and Japanese both had two types of test. While the ratio of correct answers of prefectures is available for each type of test, we use the average. As argued widely, the average scholastic ability of students seems to differ between prefectures. Furthermore, the differences occur not only in urban areas but also in the provinces. There also seems to be a close relationship between prefectures and the ranking of achievement. For example, while in both 2007 and 2008, Akita, Fukui, and Toyama prefectures were in the top five of elementary school achievement, Okinawa and Hokkaido prefectures were in the bottom two. Moreover, while the top five junior high schools included Akita, Fukui, and Toyama prefectures in both years, the bottom five included Okinawa, Hokkaido, and Osaka prefectures. Osaka prefecture was also in the bottom six elementary schools.

Using Spearman's rank correlation, we first examine the relationship between the rankings and prefectures. The results are shown in Table I. The correlations of both elementary school achievement and junior high school achievement for 2007 and 2008 were highly significantly positive. The correlation between elementary school and junior high school achievement is also significantly positive in each year. Therefore, the rankings were stable between these two years.

Why do differences between prefectures exist? What factors affect achievement? We assume that achievement is determined by the home environment and school environment. We first consider the home environment. Because elementary school and junior high school education are compulsory, all children receive it equally. However, because of borrowing constraints, students receive private school education only when their parents can afford private education. Thus, private educational expenditure, which depends on family income, affects achievement. We also use the ratio of households with members aged 65 years and over as a measure of the home environment. As double-income households have increased, it is difficult for parents to help their children with their lessons at home. Self-study is also difficult for children. However, grandparents can help and watch their grandchildren when they live together with their grandchildren.

We use the ratio of people having competed college or university. Although there are no data about the education level of parents, the ratio of people having completed college or university may be positively correlated with their education level. Children benefit from their parent's education level because of parental teaching. Moreover, parents with high education levels are eager to improve the education level of their children.[2] We consider the number of colleges and universities per one hundred thousand persons as an externality. When colleges and universities exist in a neighborhood, it may raise the cultural level of a community and work as an

[2]Kariya (2001) and Kikkawa (2006) found that the relationship between educational attainments of parents and their children has recently strengthened.

incentive to study.

Next, we consider the school environment. Education depends heavily on the human capital stock of teachers. When the human capital stock of teachers is high and the number of students per teacher is small, the student receives a good quality education. Thus, we examine both the quantity and quality of teachers.[3] However, there are no data with measures of the quality of teachers. When teachers are skillful in teaching, their classes have a good atmosphere and they can decrease the ratio of long-term absentees from school. Thus, we use the ratio of long-term absentees from school as a proxy of teacher quality.

The ratio of female workers has been increasing in many occupations. The average ratio of female teachers in elementary schools exceeds 60%. However, the ratio in junior high schools is less than half. Because experience is required to become a skilled teacher, its experience has an important influence on the achievement of students. The ratio of young teachers that are female becomes high in junior high schools. A high ratio of female teachers implies a high ratio of young teachers and a low ratio of teachers who have experience. Thus, we examine the ratio of female teachers as a proxy of experience. Note that we do not suggest that there is any difference in teaching ability between male and female teachers. We also examine

[3] By using a production function of education, Nakajima and Nakamura (2008a,b) and Nakamura and Nakajima (2009) investigated how the price of education affects inequality. In their models, diminishing returns of teachers are crucial to yield income inequality because of an increasing education price.

public education expenditure.

The test score is assumed to be subject to the following function:

$$Y = f(FI, MEM, CMP, UNI, NUM, ABS, FMT, GE), \tag{1}$$

where Y is the test score, FI is family income, MEM is the ratio of households with members aged 65 years and over, CMP is the ratio of people having competed college or university, UNI is the number of colleges and universities per one hundred thousand persons, NUM is the number of students per teacher, ABS is the ratio of long-term absentees from school (30 days or more in a school year), FMT is the ratio of female teachers, and GE is government school expenditure.

We expect the following signs of the effects:

$$\frac{\partial Y}{\partial FI} > 0, \quad \frac{\partial Y}{\partial MEM} > 0, \quad \frac{\partial Y}{\partial CMP} > 0, \quad \frac{\partial Y}{\partial UNI} > 0,$$
$$\frac{\partial Y}{\partial NUM} < 0, \quad \frac{\partial Y}{\partial ABS} > 0, \quad \frac{\partial Y}{\partial FMT} < 0, \quad \frac{\partial Y}{\partial GE} > 0. \tag{2}$$

In our regression, we test the significance of the estimates based on Eq.(2).

3. An econometric analysis of achievement

3.1. Achievement of elementary education

We investigate Eq.(1) using the scores of elementary schools. All explanatory variable data are taken from the Statistical Bureau and the Director General for Policy Planning. While we examine the test score data in 2007 and 2008, we use the same explanatory variables because of data availability. We estimate the following log-linear equation of elementary school achievement:

$$\ln Y_{i,t}^{e} = \beta_{ct} + \beta_{fi} \ln FI_i + \beta_{mem} \ln MEM_i + \beta_{cmp} \ln CMP_i + \beta_{uni} \ln UNI_i$$

$$+\beta_{num} \ln NUME_i + \beta_{abs} \ln ABSE_i + \beta_{fmt} \ln FMTE_i + \beta_{ge} \ln GEE_i + \epsilon_{i,t}, \qquad (3)$$

where $i = 1, 2, \cdots, 47$, and $t = 07, 08$. (07 and 08 represent 2007 and 2008, respectively.) $Y_{i,t}^{e}$ is the elementary school test score in prefecture i in year t, FI_i is the monthly income per household (workers households) in prefecture i in 2006, MEM_i is the ratio of households with members aged 65 years and over in prefecture i in 2005, CMP_i is the ratio of people having completed college or university in prefecture i in 2000, UNI_i is the number of colleges and universities per one hundred thousand persons in prefecture i in 2006, $NUME_i$ is the number of elementary school pupils per teacher in prefecture i in 2006, $ABSE_i$ is the ratio of long-term absentees from elementary school (30 days or more in a school year) per one thousand pupils in prefecture i in 2005, $FMTE_i$ is the ratio of elementary school teachers that are female in prefecture i in 2006, GEE_i is the public expenditure (including expenditure of municipality and prefecture) for elementary school per pupil of prefecture i in 2005, and $\epsilon_{i,t}$ is an error term with a mean of zero, which is mutually independent of all i and t.

Assuming that $\epsilon_{i,t} \sim N(0, \sigma_t^2)$, we first estimate Eq.(3) separately for 2007 and 2008 using ordinary least squares (OLS) estimation. Columns 1 and 2 of Table II show the results for 2007 and 2008, respectively. We first consider the results for 2007. The coefficient for family income is positively significant. When family income level is high, children can receive a large amount of supplementary private education. The ratio of households with members aged 65 years and over is highly positively significant. This ratio, for example, is high in the Akita prefecture, and low in the Okinawa prefecture. Grandparents living together with their grandchildren have a good effect on achievement because they can help their grandchildren. The ratio of people having completed either college or university is also positively significant. Children can benefit from the educational levels of their parents. Although the number of colleges and universities per one hundred thousand persons is positive, it is not significant. That is, it did not affect the achievement of elementary schools as an externality.

The effect of the number of elementary school pupils per teacher is close to zero. While an increase in the number of pupils would make teaching difficult, it also would enhance competition among students. These two effects will cancel each other out. Therefore, a small variation from the optimal class size would not be much of a problem.[4] However, the ratio of long-term absentees from elementary school is highly negatively significant. When a teacher is skillful, the atmosphere of a class improves and the number of long-term absentees decreases. This positively affects the achievement of pupils. The ratio of female teachers has no affect. Because the ratio of female teachers is high regardless of age, this ratio does not suggest the experience of teachers. School expenditure per pupil does not contribute to achievement. We obtain similar results for the significance of explanatory variables

[4] Oshio (2003) explained that the optimal class size depends on certain conditions.

for 2008. The constant term for 2008 is smaller than that of 2007. The test would be more difficult for 2008 than for 2007. As the adjusted R^2 indicates, the fit of Eq.(3) is better for 2007 than for 2008.

We use Ramsey's (1969) regression specification error test [RESET] which tests for a nonzero mean in the error term. The RESET statistic under the null hypothesis of no missing explanatory variables is distributed as $F_{p-1,n-k-p+1}$, where p is the maximum power of the estimated dependent variable in the original regression, raised in an auxiliary regression, n is the sample size, and k is the number of explanatory variables in the original regression. We examine $p = 3$. While the RESET test is weakly rejected for 2007, it is not rejected for 2008.[5]

Next, pooling the data for 2007 and 2008, we estimate Eq.(3) using OLS estimation. Because the difficulty of the student tests could be different between the two years, we use different constant terms. We first assume no structural change among the parameters of the explanatory variables:

$$\begin{pmatrix} \mathbf{y_{07}} \\ \mathbf{y_{08}} \end{pmatrix} = \begin{pmatrix} \iota & \mathbf{0} & \mathbf{X} \\ \mathbf{0} & \iota & \mathbf{X} \end{pmatrix} \begin{pmatrix} \mathbf{B}_\iota \\ \mathbf{B} \end{pmatrix} + \begin{pmatrix} \epsilon_{\mathbf{07}} \\ \epsilon_{\mathbf{08}} \end{pmatrix}, \tag{4}$$

where

$$\mathbf{B}_\iota \equiv \begin{pmatrix} \beta_{c07} & \beta_{c08} \end{pmatrix}', \quad \mathbf{B} \equiv \begin{pmatrix} \beta_{fi} & \beta_{mem} & \dots & \beta_{ge} \end{pmatrix}',$$

[5] When the residuals are sorted by the score, they have a tendency to decrease with an increase in the score. Thus, we added dummy variables for the top five and bottom five prefectures. While they were significant, the conclusion does not change. However, the RESET test is still rejected.

where

$$\mathbf{y_t} \equiv \begin{pmatrix} \ln Y^e_{1,t} \\ \ln Y^e_{2,t} \\ \vdots \\ \ln Y^e_{47,t} \end{pmatrix}, \quad \mathbf{X} \equiv \begin{pmatrix} \ln FI_1 & \ln MEM_1 & \ldots & \ln GEE_1 \\ \ln FI_2 & \ln MEM_2 & \ldots & \ln GEE_2 \\ \ldots & \ldots & \ldots & \ldots \\ \ln FI_{47} & \ln MEM_{47} & \ldots & \ln GEE_{47} \end{pmatrix},$$

and ι is a 47×1 vector in which all elements are 1. We assume that $\epsilon_\mathbf{t} \sim N(\mathbf{0}, \sigma^\mathbf{2}\mathbf{I})$. $\mathbf{I}$ is a 94×94 identity matrix.[6]

While we use the same explanatory variables for 2007 and 2008 because of the data availability, we investigate the properties of the estimators in the appendix. The sums of the estimators of the constant terms for the pooled sample and for the individual year samples are equal. The estimators of explanatory variables obtained from the pooled sample are equal to the averages of those obtained from the individual year samples. Furthermore, given the true variance of the error terms, the variances of the estimators are smaller for the pooled sample than for the individual year samples.

The results of the pooled sample are shown in column 3 of Table II. Family income, ratio of households with members aged 65 years and over, and ratio of people having completed university are highly significantly positive. The number of universities is significant. It might operate as an externality. The quality of teachers, but not the quantity is important for achievement. We confirm the significant difference between the constant terms. Thus, the test was more difficult in 2008 than

[6] We used boldface for vectors and matrices.

in 2007. The adjusted R^2 is higher than those for 2007 and 2008. The RESET test is not rejected. We test for structural change among the parameters of the explanatory variables, β between 2007 and 2008. The Chow (1960) test shows no structural change in the parameters between the two years. That is, the effects of the explanatory variables on achievement were stable.

Using the Goldfeld and Quandt (1965) statistic, we test the equality of variances of the error terms between 2007 and 2008. The statistic is distributed as $F_{47-9,47-9}$ under homoscedasticity and equals 2.20, which is rejected at the 5% level of significance. This implies different variances between these two years. Thus, allowing for different variances, we estimate Eq.(4) by applying the Berndt, Hall, Hall, and Hausman method of maximum likelihood (ML) estimation. By using the likelihood ratio (LR) test, we examine structural change in the explanatory variables. Column 4 of Table II shows the results. We find that homoscedasticity in the error terms does not affect the conclusion.[7]

3.2. Achievement of junior high school

We estimate the following log-linear equation of junior high school achievement:

$$\ln Y_{i,t}^{j} = \beta_{ct} + \beta_{fi} \ln FI_i + \beta_{mem} \ln MEM_i + \beta_{cmp} \ln CMP_i + \beta_{uni} \ln UNI_i$$

$$+\beta_{num} \ln NUMJ_i + \beta_{abs} \ln ABSJ_i + \beta_{fmt} \ln FMTJ_i + \beta_{ge} \ln GEJ_i + \epsilon_{i,t}, \qquad (5)$$

[7]Even when we use White's (1980) heteroscedasticity consistent estimators, the results do not change.

where $i = 1, 2, \cdots, 47$, and $t = 07, 08$. $Y^j_{i,t}$ is the junior high school test score of prefecture i in year t, $NUMJ_i$ is the number of junior high school students per teacher in 2006, $ABSJ_i$ is the ratio of long-term absentees from junior high school (30 days or more for a school year) per one thousand students in prefecture i in 2005, $FMTJ_i$ is the ratio of female teachers in junior high schools in prefecture i in 2006, GEJ_i is the public expenditure (including expenditure of municipality and prefecture) for junior high school per student of prefecture i in 2005, and $\epsilon_{i,t}$ is an error term that has a mean of zero and is mutually independent of all i and t.

Assuming that $\epsilon_{i,t} \sim N(0, \sigma_t^2)$, we first estimate Eq.(5) in both 2007 and 2008 using OLS estimation. Columns 1 and 2 of Table III show the results for 2007 and 2008, respectively. We first consider the results for 2007. Family income is highly significantly positive. Supplementary private education is more important for junior high school students than for elementary school pupils because learning becomes more difficult. The ratio of households with members aged 65 years and over is also highly significantly positive. Grandparents living together with their grandchildren are important for achievement even for junior high school students. The ratio of people having completed up to university is positively significant. While the number of universities is positively significant, it is increasingly effective on achievement as the education level of students increases.

The quality of teachers represented by the ratio of long-term absentees from junior high school is important for achievement. Although the quantity of teachers represented by the number of junior high school students per teacher is positive, it is not significant. The ratio of female teachers is negatively significant. The ratio of female teachers is high for young teachers who do not have significant experience. This result implies that experience in teaching is important for student achievement. School expenditure per student does not contribute to achievement at all. We obtain similar results for the significance of explanatory variables in 2008. While the adjusted R^2 is similar between the two years, they are higher than those of regressions of elementary school achievement. That is, the explanatory variables explain the variation in the achievement score of junior high school students better than that of elementary school pupils.

The RESET test is rejected in both years. When the residuals were sorted by test score, the residuals of Okinawa prefecture had large negative values in both the years. When we used the dummy variable only for Okinawa prefecture, numerical convergence of the estimator was not obtained in the maximum likelihood estimation. Thus, the dummy variable is considered for the bottom two, Okinawa and Hokkaido prefectures. Note that even when we used a dummy variable for Okinawa prefecture, but not including Hokkaido prefecture, the conclusion obtained from OLS estimation does not change. Eq.(5) is rewritten as:

$$\ln Y_{i,t}^{j} = \beta_{ct} + \beta_{dt} dummy_t + \beta_{fi} \ln FI_i + \beta_{mem} \ln MEM_i + \beta_{cmp} \ln CMP_i + \beta_{uni} \ln UNI_i$$

$$+\beta_{num} \ln NUMJ_i + \beta_{abs} \ln ABSJ_i + \beta_{fmt} \ln FMTJ_i + \beta_{ge} \ln GEJ_i + \epsilon_{i,t}, \qquad (6)$$

where $i = 1, 2, \cdots, 47$, and $t = 07, 08$; $dummy_t$ is the dummy variable for Hokkaido and Okinawa prefectures.

Columns 3 and 4 of Table III show the results for 2007 and 2008, respectively. We first consider the results for 2007. While the dummy variable for Okinawa and Hokkaido prefectures is highly significantly negative, the other results are similar to those not including the dummy variable. By including the dummy variable, the RESET test is not rejected and the adjusted R^2 increases. The significant dummy variable implies an own negative effect on achievement in the Okinawa and Hokkaido prefectures. While the number of universities is not significant, the results for 2008 were similar to those for 2007. The dummy variable is highly significantly negative. The RESET test is not rejected.

Next, allowing different constant terms for 2007 and 2008 and not assuming structural change among the explanatory variables between the two years, we use the pooled sample to estimate Eq.(6). While we use the dummy variable for Okinawa and Hokkaido prefectures, the effect is assumed to be the same between the two years. That is, we have $\beta_{d07} = \beta_{d08} \equiv \beta_d$.[8] The results are shown in column 5 of Table III. Family income, ratio of households with members aged 65 years and over, and ratio of people having completed university are significantly positive. The number of universities is also significantly positive. Although the quality of teachers is significant, the quantity of teachers is not significant. The ratio of female teachers is significantly negative. The dummy variable for Okinawa and Hokkaido prefectures is highly negatively significant. Because the constant term for 2008 is smaller than that for 2007, the test was more difficult in 2008 than in 2007. The

[8] We confirm that β_{dt} is not different between the two years.

adjusted R^2 is higher than those for 2007 and 2008. The RESET test is not rejected. The Chow (1960) statistic, which tests structural change among the parameters of the explanatory variables between 2007 and 2008, implies that the effects of the explanatory variables on achievement are stable.

We tested for homoscedasticity of the error terms variances between 2007 and 2008. The Goldfeld and Quandt (1965) statistic is 1.94. While it is distributed as $F_{47-10,47-10}$ under homoscedasticity, it is rejected at the 5% level of significance. Thus, allowing for different variances, we estimate Eq.(6) using ML estimation. Column 6 of Table III shows the results. We find that homoscedasticity of the variance of the error terms does not affect the results.

4. Concluding remarks

Using prefectural data of achievement test scores in 2007 and 2008, we empirically examined what factors affect the achievement of elementary school pupils and junior high school students. Our regression analysis identified some crucial factors. Family income and parental education level positively affect achievement. Grandparents living together with their grandchildren also influence achievement positively. Therefore, an increase in the ratio of family nuclei households is not good for achievement. In addition, the quality of teachers, but not the quantity, was important. No contribution of school expenditure was found. We also found a negative effect of the ratio of female teachers on junior high school achievement because the ratio of female teachers is high in young teachers.

It is difficult for the government to increase the ratio of households living with grandparents using policy. However, the government should try to raise the quality of teachers in the process of education reform. Furthermore, the government should be reminded that the contribution of public education expenditure on achievement is small.

Appendix

This appendix investigates the relationship of the estimators between the two years and the pooled samples. The OLS estimator of Eq.(4) using the pooled sample is written as:

$$\begin{pmatrix} \hat{\mathbf{B}}_\iota \\ \hat{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} \mathbf{A^{11}} & \mathbf{A^{12}} \\ \mathbf{A^{21}} & \mathbf{A^{22}} \end{pmatrix} \begin{pmatrix} \iota' \mathbf{y_{07}} \\ \iota' \mathbf{y_{08}} \\ \mathbf{X}'(\mathbf{y_{07}} + \mathbf{y_{08}}) \end{pmatrix}, \qquad (A1)$$

where

$$\mathbf{A^{11}} = \frac{1}{n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{n^2}\begin{pmatrix} \iota'\mathbf{X} \\ \iota'\mathbf{X} \end{pmatrix} \frac{1}{2}(\mathbf{X}'\mathbf{X} - \frac{1}{n}\mathbf{X}'\iota\iota'\mathbf{X})^{-1} \begin{pmatrix} \mathbf{X}'\iota & \mathbf{X}'\iota \end{pmatrix},$$

$$\mathbf{A^{12}} = -\frac{1}{n}\begin{pmatrix} \iota'\mathbf{X} \\ \iota'\mathbf{X} \end{pmatrix} \frac{1}{2}(\mathbf{X}'\mathbf{X} - \frac{1}{n}\mathbf{X}'\iota\iota'\mathbf{X})^{-1} = \mathbf{A^{21}}',$$

$$\mathbf{A^{22}} = \frac{1}{2}(\mathbf{X}'\mathbf{X} - \frac{1}{n}\mathbf{X}'\iota\iota'\mathbf{X})^{-1}.$$

Given the true variance of error terms, the covariance matrix of the estimator written in Eq.(A1) becomes:

$$\sigma^2 \begin{pmatrix} \mathbf{A^{11}} & \mathbf{A^{12}} \\ \mathbf{A^{21}} & \mathbf{A^{22}} \end{pmatrix}. \qquad (A2)$$

The OLS estimator of Eq.(3) using each year's sample, on the other hand, is written as:

$$\begin{pmatrix} \tilde{\beta}_{ct} \\ \tilde{\mathbf{B}}_\mathbf{t} \end{pmatrix} = \begin{pmatrix} D^{11} & \mathbf{D^{12}} \\ \mathbf{D^{21}} & \mathbf{D^{22}} \end{pmatrix} \begin{pmatrix} \iota'\mathbf{y_t} \\ \mathbf{X}'\mathbf{y_t} \end{pmatrix}, \qquad (A3)$$

where $t = 07, 08$, and $\tilde{\mathbf{B}}_\mathbf{t}$ represents the estimator of $\mathbf{B}$ estimated by using $\mathbf{y_t}$. In addition, we have:

$$D^{11} = \frac{1}{n} + \frac{1}{n^2}\iota'\mathbf{X}(\mathbf{X}'\mathbf{X} - \frac{1}{n}\mathbf{X}'\iota\iota'\mathbf{X})^{-1}\iota'\mathbf{X},$$

$$\mathbf{D^{12}} = -\frac{1}{n}\iota'\mathbf{X}(\mathbf{X}'\mathbf{X} - \frac{1}{n}\mathbf{X}'\iota\iota'\mathbf{X})^{-1} = \mathbf{D^{21}}',$$

$$\mathbf{D^{22}} = (\mathbf{X}'\mathbf{X} - \frac{1}{n}\mathbf{X}'\iota\iota'\mathbf{X})^{-1}.$$

The covariance matrix of the estimator written in Eq.(A3) becomes:

$$\sigma^2 \begin{pmatrix} D^{11} & \mathbf{D^{12}} \\ \mathbf{D^{21}} & \mathbf{D^{22}} \end{pmatrix}. \qquad (A4)$$

Using Eqs.(A1) and (A3), we have the following relationships between the estimators:

$$\tilde{\beta}_{c07} + \tilde{\beta}_{c08} = \hat{\beta}_{c07} + \hat{\beta}_{c08}, \qquad (A5)$$

$$\frac{1}{2}(\tilde{\mathbf{B}}_\mathbf{07} + \tilde{\mathbf{B}}_\mathbf{08}) = \hat{\mathbf{B}}. \qquad (A6)$$

That is, the sum of the estimators of the constant terms obtained from samples for each year is equal to that obtained from the pooled sample. The average of the estimators of the explanatory variables obtained from each year's samples is equal to the estimators obtained from the pooled sample.

Next, using Eqs.(A2) and (A4), we have the following relationships between the covariance matrices of the estimators:

$$Var(\tilde{\beta}_{ct}) > Var(\hat{\beta}_{ct}), \tag{A7}$$

$$Var(\tilde{\mathbf{B}}_{\mathbf{t}}) > Var(\hat{\mathbf{B}}), \tag{A8}$$

where $t = 07, 08$.

That is, the variance of the constant term for each year's sample is smaller than that of the pooled sample. Moreover, the covariance matrix of the explanatory variable estimators of the pooled sample is also smaller than those of each year's samples. The results shown in Eqs.(A5), (A6), (A7), and (A8) come from that because of limited data availability, we use the same explanatory variables for 2007 and 2008.

Table I

Spearman's Rank Correlation Coefficient

	$E07$	$E08$	$J07$	$J08$
$E07$	1			
$E08$	0.930***	1		
$J07$	0.625***	0.578***	1	
$J08$	0.665***	0.620***	0.941***	1

*Note.*E07 and E08 are the rankings of elementary school achievement in 2007 and 2008, respectively. J07 and J08 are the rankings of junior high school achievement in 2007 and 2008, respectively. $*, **, ***$ represent significance at the 10, 5 and 1% levels, respectively. The sample size is 47.

Table II

Estimation of Elementary School Achievement

	(1)07	(2)08	(3)$pool$	(4)ML
$\hat{\beta}_{fi}$	0.0735	0.105	0.0892	0.0832
	(1.95*)	(1.88*)	(2.74***)	(2.01**)
$\hat{\beta}_{mem}$	0.155	0.165	0.160	0.158
	(3.38***)	(2.44**)	(4.05***)	(3.04***)
$\hat{\beta}_{cmp}$	0.0727	0.0697	0.0712	0.0717
	(2.89***)	(1.87*)	(3.28***)	(3.24***)
$\hat{\beta}_{uni}$	0.0134	0.0206	0.0170	0.0156
	(1.27)	(1.32)	(1.87*)	(1.85*)
$\hat{\beta}_{num}$	0.0143	0.0707	0.0425	0.0316
	(0.20)	(0.67)	(0.69)	(0.42)
$\hat{\beta}_{abs}$	-0.0438	-0.0574	-0.0506	-0.0480
	(-2.88^{***})	(-2.54^{**})	(-3.84^{***})	(-3.05^{***})
$\hat{\beta}_{fmt}$	-0.0259	0.0231	-0.00142	-0.0109
	(-0.50)	(0.30)	(-0.03)	(-0.21)
$\hat{\beta}_{ge}$	-0.0289	0.0192	-0.00488	-0.0142
	(-0.44)	(0.20)	(-0.09)	(-0.21)

	(1)07	(2)08	(3)$pool$	(4)ML
$\hat{\beta}_{c07}$	4.358		4.048	4.168
	(7.12***)		(7.65***)	(6.91***)
$\hat{\beta}_{c08}$		3.551	3.860	3.980
		(3.91***)	(7.30***)	(6.59***)
$\hat{\sigma}^2_{07}(\hat{\sigma}^2)$	0.0225^2		0.0275^2	0.0204^2
$\hat{\sigma}^2_{08}$		0.0333^2		0.0306^2
$\bar{R}^2(L.L.F.)$	0.469	0.404	0.927	299.8
$RESET$	2.96*	1.12	1.41	4.44
$Chow$			0.32	3.06

Note. ˆ are the estimates. The number in () is the t-value. $*, **, ***$ represent significance at the 10, 5 and 1% levels, respectively. While RESET is distributed as $F_{2,n-k-2+1}$ under the assumption of no missing explanatory variables, it is distributed as a χ^2 distribution with two degrees of freedom for the LR test. While the Chow test is distributed as $F_{8,94-18}$ under the assumption of no structural change, it is distributed as a χ^2 distribution with eight degrees of freedom for the LR test. $\bar{R}^2$ is the adjusted R^2 and L.L.F. is the maximum log of the likelihood function. The sizes of the individual year and pooled samples are 47 and 94, respectively.

Table III

Estimation of Junior High School Achievement

	(1)07	(2)08	(3)07	(4)08	(5)$pool$	(6)ML
$\hat{\beta}_{fi}$	0.0904	0.0935	0.0810	0.0858	0.0834	0.0826
	(2.32^{**})	(2.04^{**})	(2.74^{***})	(2.08^{**})	(3.45^{***})	(3.52^{***})
$\hat{\beta}_{mem}$	0.244	0.286	0.159	0.217	0.188	0.179
	(4.80^{***})	(4.78^{***})	(3.83^{***})	(3.74^{***})	(5.53^{***})	(4.94^{***})
$\hat{\beta}_{cmp}$	0.0700	0.0878	0.0387	0.0621	0.0504	0.0466
	(2.42^{**})	(2.58^{**})	(1.71^{*})	(1.97^{*})	(2.72^{**})	(2.21^{**})
$\hat{\beta}_{uni}$	0.0255	0.0236	0.0180	0.0176	0.0178	0.0179
	(2.14^{**})	(1.69^{*})	(1.97^{*})	(1.38)	(2.38^{**})	(2.22^{**})
$\hat{\beta}_{num}$	0.0941	0.120	0.0764	0.106	0.0911	0.0863
	(1.04)	(1.14)	(1.12)	(1.11)	(1.63)	(1.35)
$\hat{\beta}_{abs}$	−0.0827	−0.0796	−0.0628	−0.0634	−0.0631	−0.0630
	(-2.83^{***})	(-2.32^{**})	(-2.80^{***})	(-2.03^{**})	(-3.44^{***})	(-3.16^{***})
$\hat{\beta}_{fmt}$	−0.121	−0.123	−0.0569	−0.0705	−0.0637	−0.0615
	(-2.67^{**})	(-2.31^{**})	(−1.56)	(−1.39)	(-2.14^{**})	(-1.70^{*})
$\hat{\beta}_{ge}$	−0.0581	−0.0474	−0.0011	−0.0006	−0.0009	−0.0009
	(−0.86)	(−0.60)	(−0.02)	(−0.01)	(−0.02)	(−0.02)

	(1)07	(2)08	(3)07	(4)08	(5)$pool$	(6)ML
$\hat{\beta}_{c07}$	4.435		3.989		3.981	3.984
	(6.70***)		(7.84***)		(9.57***)	(7.96***)
$\hat{\beta}_{c08}$		4.191		3.825	3.834	3.836
		(5.38***)		(5.39***)	(9.22***)	(7.66***)
$\hat{\beta}_{d07}(\hat{\beta}_d)$			−0.0922		−0.0840	−0.0866
			(−5.38***)		(−6.00***)	(−5.14***)
$\hat{\beta}_{d08}$				−0.0757		
				(−3.17***)		
$\hat{\sigma}^2_{07}(\hat{\sigma}^2)$	0.0236^2		0.0179^2		0.0207^2	0.0160^2
$\hat{\sigma}^2_{08}$		0.0277^2		0.0249^2		0.0224^2
$\bar{R}^2(L.L.F.)$	0.666	0.631	0.808	0.702	0.942	326.0
$RESET$	17.61***	8.00***	1.20	1.25	0.25	4.57
$Chow$					0.13	1.32

Note. ˆ are the estimates. The number in () is the t-value. $*, **, ***$ represent significance at the 10, 5 and 1% levels, respectively. While RESET is distributed as $F_{2,n-k-2+1}$ under the assumption of no missing explanatory variables, it is distributed as a χ^2 distribution with two degrees of freedom for the LR test. While the Chow test is distributed as $F_{8,94-19}$ under the assumption of no structural change, it is distributed as a χ^2 distribution with eight degrees of freedom for the LR test. $\bar{R}^2$ is the adjusted R^2 and L.L.F. is the maximum log of the likelihood function. The sizes of the individual year and pooled samples are 47 and 94, respectively.

References

[1] Chow, G. C. (1960). Tests of equality between sets of coefficients in linear regressions, *Econometrica* **28**, 591–605.

[2] Goldfeld, S. M. and Quandt, R. E. (1965). Some tests for homoscedasticity, *J. Amer. Stat. Assoc.* **60**, 539–547.

[3] Kariya, T. (2001). *Classed Japan and crisis of education: From reproduction of inequality to a society of incentive-divide*, (In Japanese), Tokyo: Yushindokoubunsha.

[4] Kariya, T. (2007). *Illusion of education reform*, (In Japanese), Tokyo: Chikumashobo.

[5] Kikkawa, T. (2006). *Education and social inequality: Contemporary educational credentialism in Japan*, (In Japanese), Tokyo: University of Tokyo Press.

[6] Nakajima, T. and Nakamura, H. (2008a). The price of education and inequality, Mimeo, Osaka City University.

[7] Nakajima, T. and Nakamura, H. (2008b). A note on significant effects of education on inequality, Mimeo, Osaka City University.

[8] Nakamura, H. and Nakajima, T. (2009). Education and inequality, Mimeo, Osaka City University.

[9] Nishimura, K. (2003). Evaluation of 'yutori kyouiku' by economics. (In Japanese), in Ito, T. ed., *Kyouikukaikaku no keizaigaku*, Tokyo: Nihonkeizai-hyouronsha.

[10] Oshio, T. (2003). *Investigation of education by economics*, (in Japanese), Tokyo: Nihonhyoronsha.

[11] Ramsey, J. B. (1969). Tests for specification errors in classical linear squares regression observations, *J. Roy. Stat. Soc.* **B2**, 350–371.

[12] White H. (1980). A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity, *Econometrica* **48**, 817–838.