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# Discussion Paper Series

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# Education and Inequality*

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**Abstract:**

We try to explain, in theoretical terms, Japan's increasing income inequality in terms of higher education. By examining the relationship between the cost of education and the wage rate, we explain how the link between parents' and their children's educational advancement is strengthened. Given that the wage rate does not increase much, even if the initial education level of the poor is higher than the poverty trap threshold, they cannot attain higher education in the long run because of an increase of the cost of education. A sufficient subsidy for education scholarships can enhance income equality in the long run.

**Keywords:** Education price, Wage rate, Income inequality

**JEL Classification:** I20, O11, O15.

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## 1. Introduction

When Japan experienced rapid economic growth from 1955 to 1973, it was widely argued that Japan had successfully escaped from the poverty experienced after World War II and the entire population of one hundred million belonged to the middle class. However, economic growth has since slowed and a large increase in income inequality has been recently identified. Using the *Income Redistribution Survey* data reported by the Ministry of Health, Labour and Welfare, Tachibanaki (2005) found that the Gini coefficient measured by redistributed income indicated that income inequality increased from 0.314 in 1972 to 0.381 in 2002. He investigated changes in wages, changes to the household structure, and the roles of taxes and social security systems to explain increasing inequality. Ohtake (2005), on the other hand, found that while the Gini coefficients within age groups changed little, elderly persons in which its coefficient was relatively high increased. Thus, he concluded that the aging society could explain a large part of the increase in inequality. However, examining the *National Survey of Family Income and Expenditure* in years 1999 and 2004 reported by the Statistical Bureau and the Director-General for Policy Planning, Uni (2008) found that the Gini coefficients within the age groups other than elderly groups increased. He considered that increasing inequality in wages caused the recent increase in income inequality.[1] Therefore, causes of rinsing income inequality are still controversial and merit further research.

[1]He also used the *Employment Status Survey* data and the micro data provided by Institute of Economic Research, Hitotsubashi University.

As shown in Galor and Zeira (1993), given capital market imperfections, educational opportunities play a crucial role in income inequality. How then has educational attainment been changing over time in Japan? The rate of advancement to upper secondary schools reached ninety percent in 1964. Figure 1, on the other hand, shows the rate of advancement to universities (including junior colleges) in Japan from 1963 to 2004.[2] Although the advancement rate continued to increase during the period of rapid economic growth, it has increased little since. By considering educational credentialism, Kikkawa (2006) investigated the relationship between education and social inequality and found that the relationship of educational attainments between parents and their children has recently strengthened.

How is income inequality related to this relationship? As discussed in Higuchi (1994) and Teruyama and Ito (1994), the private cost of education and the income of parents are important for explaining inequality.[3] By considering the price of education and the wage rate, the current paper tries to answer this question theoretically. Given an imperfect credit market, children can receive higher education only when

[2]All data used in Figures 1, 2, and 3 were provided by the Statistical Bureau and the Director-General for Policy Planning. Owing to the availability of data, the time-series data represent every year for the years between 1963 and 2004.

[3]Higuchi (1994) found that while the private cost of education has increased more than the consumption price index, the relationship between the advancement rate to universities and the income of parents has strengthened since the rapid-growth period. In addition, Teruyama and Ito (1994) point out the possibility that intergenerational transmission of earnings ability through educational investment could become a crucial factor that increases inequality.

their parents finance the private cost of education, including tuition fees and living expenses. Figure 2 shows the average annual family income per household (workers' households) in Japan. Family income has stagnated during the last decade because of slow economic growth although it had been increasing until then. In addition, although earnings per worker had increased more than the advancement rate to universities until the last decade, neither increased much during the past decade. Because an increase in the advancement rate can be considered to indicate an increase in the education levels of workers, we can infer that the return on education has not increased during the decade. Figure 3 shows the ratio of educational expenditure to family income. While educational expenditure includes school fees, the cost of school textbooks, and tutorial fees, school fees account for approximately eighty percent of the total. Although the expenditure to income ratio decreased during the period of rapid economic growth, it has since been increasing. Furthermore, using the data reported by the Japan Students Services Organization, we found that tuition fees in both private and public universities have been increasing. Tuition fees increasing faster than income has made educational expenditure a greater burden on the budgets of households.[4]

The current paper derives the price of education by explicitly considering the

[4] This can be observed also in the USA. The biennial report from the National Center for Public Policy and Higher Education found that college tuition fees increased 439 percent from 1982 to 2007 while median family income increased 147 percent. The New York Times (on December 3, 2008) concluded that even before the recession it had become difficult for most Americans to receive higher education because of the increasing cost.

supply side of education. Because education depends heavily on the human capital stock of teachers and both teachers and students are individuals, we assume a diminishing marginal product of teachers. This implies that the price of education rises with an increase in education level.

The income of an individual is assumed to be used for consumption or a bequest, with a bequest used for educational expenditure to benefit the individual's child. Because an individual without education can work as an unskilled worker, we allow individuals to have zero expenditure on education. Thus, following Moav (2002) and Galor and Moav (2004, 2006), we assume a convex bequest function. Furthermore, assuming that a rise in the average human capital of an economy increases labor productivity, we simply take into account the wage rate. When the external effect is strong, the wage rate increases greatly with the accumulation of average human capital. However, when the external effect is weak, the wage rate does not increase much with the accumulation of average human capital.

If the wage rate increases sufficiently, even if the initial education level of the poor is zero-i.e., even if the poor are initially caught in a poverty trap-they can escape from the trap sooner or later because of the accumulation of human capital by the rich. In this case, while the education price increases, the education level of the poor increases and both rich and poor eventually attain the same level of education. Income inequality then disappears. This result would correspond with the findings regarding the increasing educational attainment and the decreasing income inequality that occurred during the period of rapid economic growth.

However, if the wage rate does not increase much, the poor cannot escape the trap. Furthermore, even if the initial education level of the poor is higher than the threshold of the poverty trap, they cannot receive education in the long run because the education price increases more rapidly than their income. Although the education levels of both rich and poor will increase in early periods, the education level of the poor will start to decrease sooner or later. This means that at first an increase in income inequality would not be much of a problem, but it would certainly become serious later. Therefore, we can explain the recent increase in income inequality and the close relationship regarding educational attainments between parents and their children.

We also investigate policies for attaining income equality and a higher GDP. Incorporating externalities that become the engine of growth, Glomm and Ravikumar (1992), Zhang (1996), and Bräuninger and Vidal (2000) investigated the effects of various policies, such as public versus private education, on equality and economic growth. Given the assumptions of local externalities and an economy-wide externality, Bénabou (1996) studied the effects of socioeconomic stratification and alternative systems of educational finance on inequality and growth. By considering a dynamic heterogeneous-agent economy, Bénabou (2002) investigated the effects of progressive income taxes and education finance. We investigate the effects of certain education policies on inequality and macroeconomic development when education cost increases endogenously. We assume that consumption taxes are used to subsidize the educational institution, where the subsidy is used for scholarships or a decrease in tuition fees. If the subsidy is sufficiently large, even if the poor are initially caught in a poverty trap they can eventually attain the same education level as the rich, and macroeconomic development is therefore enhanced.

Deriving the price of education, Nakajima and Nakamura (2008a,b) also investigated the effect of the price on income inequality. While Nakajima and Nakamura (2008a) assumed a convex bequest function, Nakajima and Nakamura (2008b) assumed that the parental utility function depends on the human capital stock of children. However, in neither case did they take into account any change in the wage rate. Furthermore, the productivity of elementary education played a crucial role to explain income inequality in their models. While we can consider both the effects of the education price and the wage rate, they both are important to explain the inequality in our model. In addition, the policies suggested to enable income equality differ between these papers and the current paper. While Nakajima and Nakamura (2008a) considered transfers to students, the amount of transfer must be enough to eliminate multiple steady states. This elimination, on the other hand, is not required for the policies suggested here. Moreover, while Nakajima and Nakamura (2008b) considered lump-sum taxes to suppress the increase in the education price, i.e., education level, we consider consumption taxes to avoid suppressing the increase in education level.

The rest of the paper is organized as follows. Section 2 explains our model and Section 3 describes the effects of education on income inequality. We conclude in Section 4 with a brief summary and a few remarks.

## 2. Model

Our model is a closed overlapping-generations economy. If parents decide to spend on education, their children receive this education in the first period. In the second period, they work. The population of each generation is assumed to be $L$, a constant. We assume that the initial numbers of rich and poor are, respectively, $\lambda L$ and $(1-\lambda)L$. Their education levels are denoted as $e_{r,-1}$ and $e_{p,-1}$, respectively. We assume that $e_{r,-1} \geq e_{p,-1}$. We consider a consumption goods sector and an educational sector. Firms in the consumption goods sector are perfectly competitive. A nonprofit organization runs the institution of education.

### 2.1 Educational sector

We first describe the relationship between education and human capital formation. For simplicity, the human capital of an individual is assumed to be of the following linear type:

$$h(e_{it-1}) = 1 + e_{it-1}, \tag{1}$$

where $i = r, p$, and $h(\bar{e}, e_{rt-1})$ and $h(\bar{e}, e_{pt-1})$ are the levels of human capital stock of the rich and poor, respectively. $e_{rt-1}$ and $e_{pt-1}$ are the respective education levels of the rich and poor received in period $t-1$.

We assume that individuals have to pay the private cost for higher education. However, they can acquire labor-basic skills even in the absence of higher education.[5]

[5]These skills can be obtained through elementary education though for simplicity we do not consider it. Galor and Moav (2006) emphasized the timing of public education for the masses to

Thus, we assume a positive constant term in (1). This implies that workers even with no education can obtain income to live.

Education can be considered as an outcome of collaboration between students and teachers. We assume that individuals with the highest education level can become teachers. This implies that teachers are among the rich. The total amount of education is assumed to be produced subject to the following Cobb-Douglas production function:

$$e_{at}L_t^S = (h(e_{rt-1})L_t^T)^{\alpha}(L_t^S)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{2}$$

where $e_{at}$ is the average educational level per student that is received in period $t$, $L_t^T$ is the number of teachers in period $t$, and $L_t^S$ is the number of students in period $t$.

Equation (2) can be written as

$$e_{at} = (h(e_{rt-1})\tau_t)^{\alpha}, \tag{3}$$

where $\tau_t \equiv L_t^T / L_t^S$ is the number of teachers and students in period $t$, respectively.

When the human capital stock of teachers is high and the number of teachers per student is large, the student receives a large amount of education. Moreover, because both teachers and students are individuals, the marginal product of teachers decreases as the human capital stock of teachers increases.

Tuition is used to pay the wages of teachers. The balanced budget of this orga-

explain both inequality within a country and the process of macroeconomic development.

nization can be written as

$$p_t e_{at} L_t^S = w_t h(e_{rt-1}) L_t^T, \tag{4}$$

where $p_t$ is educational price and $w_t$ is the wage rate.

While the left-hand side in (4) means tuition, the right-hand side implies the wage cost.

The educational price is determined by a zero profit condition.[6] Using (3) and (4), the education price is represented as

$$p_t = w_t e_{at}^{(1-\alpha)/\alpha}. \tag{5}$$

The educational price becomes an increasing function with respect to the education level. Furthermore, the price increases as the wage rate increases.

## 2.2 Consumption goods sector

While the rich are employed in both the educational sector and in the consumption goods sector, the poor are only employed in the consumption goods sector. For simplicity, the production function is assumed to be of the following linear type:

$$Y_t = A_t[h(e_{pt-1})(1-\lambda)L + h(e_{rt-1})L_t^C], \tag{6}$$

where $A_t$ represents the external effect. $L_t^C$ is the number of rich employed in the consumption goods sector. The equality, $L_t^C + L_t^T = \lambda L$, holds in the labor market.

[6]By considering a model in which the outputs depend partially on customers as inputs, Rothschild and White (1995) showed that prices that charge customers for what they get on net from the firm are competitive and support efficient allocation.

We assume that the average human capital of this economy influences the productivity of labor positively:[7]

$$A_t = [\lambda h(e_{rt-1}) + (1-\lambda)h(e_{pt-1})]^{\delta}, \quad \delta \geq 0. \tag{7}$$

This externality becomes strong as parameter $\delta$ increases. The externality does not apply in the case of $\delta = 0$.

With the price of consumption goods normalized to unity, the income of a worker becomes $w_t h(e_{it-1})$. The wage rate is represented by the externality. That is, we have $w_t = A_t$, where $w_t$ is the wage rate. An increase in the average human capital of an economy implies an increase in the wage rate. When the externality is strong, the wage rate increases rapidly.

## 2.3 Individuals

Individuals live in two periods. While we consider credit market imperfections, for simplicity, we assume that individuals cannot borrow or lend; i.e., that there is no credit market. Thus, educational expenditure is possible only if parents allocate bequests to their children. Individuals work in the second period. Their income is used for consumption or a bequest. Because they can work as unskilled workers even in the case of zero educational expenditure, we allow a convex bequest function.

[7] While Galor and Moav (2000) assumed perfect substitutability between skilled labor and unskilled labor to consider composite labor, they also assumed skill-biased technological progress. In their model, technological progress raises the return to ability and generates an increase in wage inequality.

The utility maximization problem of an individual born in period $t-1$ is

$$\max_{c_{it}, e_{it}} \beta \ln c_{it} + (1-\beta)\ln(b_{it}+\theta), \qquad \theta > 0, \tag{8}$$

$$s.t. \qquad w_t h(e_{it-1}) = c_{it} + b_{it}, \tag{9}$$

$$b_{it} = p_t e_{it}, \tag{10}$$

where $i = r, p$. $c_{rt}$ and $c_{pt}$ are the consumption levels of the rich and poor, respectively. $b_{rt}$ and $b_{pt}$ are the bequest levels of the rich and poor, respectively.

The bequest is used only for educational expenditure. Parameter $\theta$ allows a zero bequest; i.e., a zero educational expenditure for children. The first-order conditions imply the demand for education:

$$p_t e_{it} = (1-\beta) w_t e_{it-1} + (1-\beta) w_t - \beta\theta. \tag{11}$$

When the vertical and horizontal axes respectively represent the amount of educational expenditure and the educational level of a parent, the intercept point is represented by $(1-\beta)w_t - \beta\theta$. When it is negative, educational expenditure becomes a convex function. In addition, the ratio of educational expenditure to income increases as the level of income increases.[8]

Using (1) and (7), we define $a(e_{at-1})$ as

$$a(e_{at-1}) \equiv (1-\beta) - \frac{\beta\theta}{(1+e_{at-1})^{\delta}}, \tag{12}$$

[8] When the intercept point is positive, on the other hand, the ratio of educational expenditure to income decreases with an increase in the income level.

where $e_{at-1} = \lambda e_{rt-1} + (1-\lambda)e_{pt-1}$; i.e., $e_{at-1}$ is the average education level of the rich and poor.

$a(e_{at-1})$ is an increasing function of $e_{at-1}$. We assume that $a(0) < 0$; i.e., $(1-\beta)-\beta\theta < 0$. The intercept point in (11) is represented by $w_t a(e_{at-1})$. Thus, the sign of $a(e_{at-1})$ is equal to that of the intercept point. The assumption of $a(0) < 0$ implies that educational expenditure becomes a convex function when the average education level is low. $a(e_{at-1})$ takes a positive value with a high average education level. In this case, the intercept point of educational expenditure also becomes positive.

## 3. Education and inequality

### 3.1 Dynamics in the case of homogeneous individuals

While this paper investigates the effect of education on income inequality, we first consider the case that there is no difference in initial educational levels of the rich and poor; i.e., the case that $e_{r,-1} = e_{p,-1}$.

Using (5), (11), and (12), the dynamics can be represented as

$$e_t^{1/\alpha} = (1-\beta)e_{t-1} + a(e_{t-1}). \tag{13}$$

Figure 4 shows the dynamics. $f(e_t)$ and $g(e_{t-1})$ respectively represent the left- and right-hand sides of (13). There exist multiple steady states in education level regardless of the strength of the externality because of a convex educational expenditure.[9] While a poverty trap that takes a value of zero is stable, $e^{**}$, the threshold

[9] If $a(0)$ were sufficiently small, there would be no intersection between $f(e_t)$ and $g(e_{t-1})$. The educational level would become zero regardless of the initial value in this case.

of the poverty trap, is unstable and $e^*$ is stable. When the initial education level $e_{-1}$ is lower than $e^{**}$ but higher than $e_d$, the education level converges to zero. Note that $g(e_d) = 0$. That is, the educational expenditure remains zero in the case where $e_{-1} \leq e_d$. However, if the initial education level is larger than the threshold of the poverty trap, it converges to the high-level steady state, $e^*$. That is, the relationship between the initial value and the threshold crucially affects the dynamics. An increase in the education level causes an increase in the education price because given the diminishing returns of teachers, both the number of teachers per student and their education level increase. The number of teachers per student is an increasing function with respect to the education level:

$$\tau_t = \frac{(1-\beta)e_{t-1} + a(e_{t-1})}{h(e_{t-1})}.$$

Let us perform a comparative statistic analysis:

$$\frac{\partial e^{**}}{\partial \alpha} < 0, \quad \frac{\partial e^{**}}{\partial \beta} > 0, \quad \frac{\partial e^{**}}{\partial \theta} > 0, \quad \frac{\partial e^{**}}{\partial \delta} < 0,$$

$$\frac{\partial e^{*}}{\partial \alpha} > 0, \quad \frac{\partial e^{*}}{\partial \beta} < 0, \quad \frac{\partial e^{*}}{\partial \theta} < 0, \quad \frac{\partial e^{*}}{\partial \delta} > 0.$$

We first investigate the effect of the parameters on $e^*$; i.e., the education level of a stable steady state. A rise in the weight of consumption in the utility function decreases the education level. A rise in the bequest threshold in the utility funciton also decreases the education level. However, any rise in the efficiency of the institution of education or the external effect increases the education level.[10]

[10] We assume that $e^{**} > 1$ to see the effect of $\alpha$.

Next, we look at the effect of the parameters on $e^{**}$; i.e., the education level of an unstable steady state. Any rise in the weight of consumption or the bequest threshold in the utility function increases the education level of an unstable steady state. However, a rise in the efficiency of the educational institution decreases the education level. When the threshold of the low-income trap decreases, it will be easier for a trapped economy to converge to the high-income steady state.

## 3.2 How does education influence inequality?

In this section, given the assumption that $e_{r,-1} > e_{p,-1}$-i.e., that the initial education level of the rich is higher than that of the poor-we investigate how education influences income inequality and macroeconomic development.

The dynamics of the average education level is represented as

$$e_{at}^{1/\alpha} = (1-\beta)e_{at-1} + a(e_{at-1}). \tag{14}$$

Even when the rich and the poor receive different levels of education, this is essentially the same as (13), which shows the dynamics of homogeneous individuals.

The dynamics of education for the rich and poor, respectively, are

$$e_{rt} = \frac{(1-\beta)e_{rt-1} + a(e_{at-1})}{[(1-\beta)e_{at-1} + a(e_{at-1})]^{1-\alpha}}, \tag{15}$$

$$e_{pt} = \frac{(1-\beta)e_{pt-1} + a(e_{at-1})}{[(1-\beta)e_{at-1} + a(e_{at-1})]^{1-\alpha}}. \tag{16}$$

The denominator in each equation represents the education price. The assumption of diminishing returns for teachers implies that the education price is determined by the weighted average of educational demand between rich and poor. The dynamics of education for the rich and poor are thus mutually dependent through the external effect and the education price.

The dynamics crucially depend on the sign of $a(e_a)$ in steady states where $a(e_a)$ determines the structure of educational expenditure. Assuming that $a(e^{**}) < 0$, we consider two cases: $a(e^{*}) > 0$ and $a(e^{*}) < 0$. The assumption of $a(e^{*}) > 0$ means that the constant term of educational expenditure is positive at the high-level steady state; i.e., parents with no education can finance education for their children at this steady state. The assumption of $a(e^{*}) < 0$, on the other hand, implies that they cannot finance it. Note that the strength of the externality represented by $\delta$ positively affects $a(e^{*})$. If the wage rate does not increase much because of a weak externality, $a(e_a)$ is still negative at the high-level steady state; i.e., $a(e^{*}) < 0$. However, if the wage rate increases sufficiently because of a strong externality, $a(e_a)$ becomes positive at the high-level steady state; i.e., $a(e^{*}) > 0$.

We first investigate the case where the wage rate increases sufficiently; i.e., that $a(e^{*}) > 0$. The phase diagram in Figure 5 shows the dynamics in this case.[11] Steady state $O$, which implies a poverty trap, is stable. Steady state $C$, which is the threshold of the poverty trap, is unstable. Steady state $B$ is stable.

Let us consider the initial point where $e_{r,-1} > e^{**}$ and $e_{p,-1} = 0$. This indicates that while the initial education level of the rich is higher than the threshold of the poverty trap, the initial education level of the poor is zero. This implies that the

[11]The appendix explains the phase diagrams in Figures 5 and 6.

education level of the poor is caught in the poverty trap. While the education level of the rich increases, the education level of the poor is temporarily caught in the poverty trap, so income inequality between the rich and the poor temporarily rises. All the students in the educational institution are children of the rich. The dynamics of the education level of the rich is the same as those of homogeneous individuals:

$$e_{rt}^{1/\alpha} = (1-\beta)e_{rt-1} + a(e_{at-1}), \tag{17}$$

where $e_{at-1} = 1 + \lambda e_{rt-1}$.

We can see the dynamics in Figure 4. Note that Figure 5 shows the dynamics in the case where both rich and poor make educational expenditures. Because the education level of the rich increases, the wage rate increases. Let us assume that $a(e_{at-1})$ becomes positive when the education level of the rich becomes greater than $e_s$, in which case $e_s$ shown in Figure 5 is defined as

$$(1-\beta) - \frac{\beta\theta}{(1+\lambda e_s)^{\delta}} = 0.$$

When the education level of the rich exceeds $e_s$, the wage rate becomes high enough to enable the poor to receive education. The education level of the poor then starts to increase from point $A$ in Figure 5. Note that point A is higher than $e_s$ because the bequest level of the poor does not start to increase until the education level of the rich exceeds $e_s$. The educational levels of the rich and poor eventually converge to $e^*$. That is, income inequality disappears in the long run.

Using (15) and (16), the difference in education levels between rich and poor is represented as

$$e_{rt} - e_{pt} = \frac{1-\beta}{p_t}(e_{rt-1} - e_{pt-1}). \tag{18}$$

This equation implies that inequality between rich and poor depends on the term, $(1-\beta)/p_t$. If its value is smaller (larger) than unity, the income inequality narrows (widens). We have the following relationship between the education price in the steady states and $a(e)$:[12]

$$\frac{1-\beta}{p} \overset{>}{\underset{<}{-}} 1 \quad \Leftrightarrow \quad a(e) \overset{<}{\underset{>}{-}} 0.$$

When $a(e) > (<) 0$ at $e = e^*$, the term $(1-\beta)/p$ is smaller (larger) than unity. Thus, income inequality narrows when $a(e^*) > 0$.

Economic development of this economy follows an inverted-U shape between the level of inequality and the level of GDP per capita. This relationship is known as the Kuznets curve (Kuznets, 1955), and implies that an increase in income inequality is a necessary ingredient for future economic growth because an increase in the education level of the rich increases the wage rate through the externality.[13]

Here, we have a proposition for the case where the wage rate increases sufficiently.

**Proposition 1:** *If the wage rate sufficiently increases to ensure* $a(e^*) > 0$, *economic development follows the Kuznets curve. Even if the poor are initially caught in a poverty trap, income inequality between the rich and the poor disappears in the long run.*

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[12]See the appendix.

[13]Assuming local home environment externality and global technological externality, Galor and Tsiddon (1997) derived the proposition of Kuznets (1955).

Next, we look at the case where the wage rate does not increase much: i.e., $a(e^*) < 0$. The phase diagram in Figure 6 shows the dynamics. Steady state $O$, which implies a poverty trap, is stable. Steady state $C$, which is the threshold of the poverty trap, is unstable. Steady state $B$ is a saddle point. Compared with Figure 5, the slopes of $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ on $e^*$ are opposite.[14] While the saddle path always exists on the 45 degree line, the dynamics become asymmetric depending on the ratio of rich and poor.

Let us assume that the initial point is located at $A_1$ in Figure 6. That is, while the initial education level of the rich is higher than the threshold of the poverty trap, the initial education level of the poor is zero. The dynamics of the education level of the rich follows (17). Their education level converges to $e^*$. However, an increase in the wage rate is insufficient to ensure a positive $a(e_{at-1})$. Therefore, the education level of the poor has been caught in the poverty trap. The income inequality between the rich and the poor monotonically increases. Compared with the case of $a(e^*) > 0$, the GDP level becomes lower because the education levels of both rich and poor become lower.

Now, we consider the case where the initial education levels of both rich and poor are higher than the threshold of the poverty trap. This can be represented by the initial point, $A_2$, in Figure 6. The education levels increase for both in the early periods. An increase in income inequality is inconspicuous. However, the

[14] The lines, $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$, become identical in the case that $a(e^*) = 0$. A continuum of steady states exists on that line.

education level of the poor, but not of the rich, decreases sooner or later because the increase in the wage rate is small and the education price increases more rapidly than the income of the poor. The education level of the poor eventually becomes zero. Increasing income inequality then clearly exists. If the number of the rich is large and their education level is high in the initial period-i.e., the inequality in the initial period is large-the education level of the poor decreases more rapidly. Given that the educational level of the poor is zero, the dynamics of the educational level of the rich follows (17).

Here, we have the following proposition for the case in which the wage rate does not increase much.

**Proposition 2:** *If the wage rate does not increase much (i.e.,* $a(e^*) < 0$*) and given that the initial education level of the rich is higher than that of the poor, income inequality between them increases in the long run regardless of the initial education level of the poor. In addition, the GDP level is lower than the GDP level in the case of* $a(e^*) > 0$*.*

## 3.3 Effects of policies on inequality

Are there effective policies to promote income equality and enhance macroeconomic development in the case of $a(e^*) < 0$? Let us consider consumption taxes that are used to subsidize the educational institution. The budget constraint on an individual noted in (9) can be rewritten as

$$w_t h(e_{it-1}) = (1 + d_t)c_{it} + b_{it}, \tag{19}$$

where $i = r, p$. $d_t c_{it}$ represents the amount of consumption taxes.

The first-order conditions of the utility maximization problem yield the consumption expenditure:

$$d_t c_{it} = \frac{d_t}{1 + d_t}(\beta w_t e_{it-1} + \beta w_t + \beta\theta), \quad i = r, p. \tag{20}$$

While consumption taxes decrease the consumption expenditure, they do not affect the educational expenditure.[15]

The tax revenue, $\lambda L d_t c_{rt} + (1 - \lambda) L d_t c_{pt}$, is used to subsidize the educational institution. While we first assume that the subsidies are used for scholarships, for simplicity the scholarships are assumed to be equally granted. The budget of the educational institution noted in (4) is rewritten as

$$p_t e_{at} L + [\lambda d_t c_{rt} + (1 - \lambda) d_t c_{pt}] L = h(e_{rt-1}) \tau_t L + SL, \tag{21}$$

where $S$ is the amount of the scholarships.

This budget is the same as that in (4) because of the equality

$$S = \lambda d_t c_{rt} + (1 - \lambda) d_t c_{pt}.$$

Given $S$, $d_t$ is set to satisfy this equation.

A constant $S$ implies that the poor benefit more than the rich because the educational expenditure of the poor is smaller than that of the rich. The educational expenditure becomes

$$p_t e_{it} = (1 - \beta) w_t e_{it-1} + (1 - \beta) w_t - \beta\theta + S. \tag{22}$$

[15] If we considered income taxes or lump-sum taxes, these taxes would decrease both the consumption and the educational expenditure.

Scholarships increases educational expenditure. When the vertical and horizontal axes respectively represent the education price and the education level, given the education price, the demand for education increases because of scholarships, and the supply of education is unchanged. Therefore, the level of education increases. A sufficient scholarship amount can make $a(e^*)$ positive:

$$a(e^*) = (1-\beta) + \frac{S-\beta\theta}{(1+e^*)^\delta} > 0. \tag{23}$$

We assume that $1-(S-\beta\theta)\delta/(1+e^*) > 0$ to ensure $\partial a(e^*)/\partial S > 0$. Note that $e^*$ is an increasing function of $S$. In this case, the wage rate can become high enough to allow the poor to spend on educational expenditure for their children. As the poor can escape from the poverty trap, income equality can be attained.[16]

We now have the following proposition regarding the effect of scholarships.

**Proposition 3:** *If (23) holds with a sufficient scholarship amount and given that the initial education level of the rich is above the poverty trap threshold, both rich and poor can eventually attain the same education level of the high-level steady state. Therefore, income inequality between them disappears.*

Next, we assume that tax revenue from the consumption tax is used to decrease the education price; i.e., tuition fees. For simplicity, we assume that the education price decreases by a constant proportion $1-g$, where $0 < g < 1$. Subsidies are used

[16] It could be possible that a further sufficient amount of $S$ makes the poverty trap disappear. In addition, we could infer that a cut in subsidies for private schools would increase income inequality and cause stagnant macroeconomic development because the price of education would increase by more than the income of the poor.

to compensate the educational institution for lost tuition revenue:

$$\lambda L d_t c_{rt} + (1-\lambda) L d_t c_{pt} = g p_t [\lambda L e_{rt} + (1-\lambda) L e_{pt}]. \tag{24}$$

Given $g$, $d_t$ is set to satisfy this budget. This policy implies that the rich benefit more than the poor because the rich spend more on their educational expenditure than do the poor.

When the vertical and horizontal axes respectively represent the education price and the education level, given the education price, the demand for education is unchanged, and the supply of education increases because of the subsidies. Individuals pay tuition fees with the education price represented by $\tilde{p}_t = (1-g) w_t e_{at}^{(1-\alpha)/\alpha}$. The dynamics of average education level written in (14) can be represented as

$$(1-g) e_{at}^{1/\alpha} = (1-\beta) e_{at-1} + a(e_{at-1}). \tag{25}$$

Because the education level increases, it is possible to make $a(e^*)$ positive. Compared to (23), this policy is felt only through the education level and there is no direct effect. Therefore, when the external effect is weak, the amount of consumption tax would become large because an increase in the education level would be small.

## 4. Conclusion

We tried to explain the recent increase in income inequality within Japan with respect to higher educational attainment. When the wage rate increases slowly, income inequality will rise because the education price increases more quickly than the income of the poor. In this case, even if the initial education level of the poor is higher than the poverty trap threshold, their education level sooner or later starts to decrease. Therefore, income inequality gradually occurs.

We also examined the effect of policies to decrease income inequality and enhance macroeconomic development. We considered the use of subsidies to provide scholarships and decrease tuition fees. When those subsidies are sufficient to enable a change from one regime having a saddle point to another having a stable steady state, it becomes possible to decrease inequality and enhance macroeconomic development.

## Appendix

We explain the dynamics that are represented by (15) and (16). We rewrite the dynamics as follows:

$$\Delta e_{rt} = \frac{(1-\beta)e_{rt-1} + a(e_{at-1})}{((1-\beta)e_{at-1} + a(e_{at-1}))^{1-\alpha}} - e_{rt-1}, \tag{A1}$$

$$\Delta e_{pt} = \frac{(1-\beta)e_{pt-1} + a(e_{at-1})}{((1-\beta)e_{at-1} + a(e_{at-1}))^{1-\alpha}} - e_{pt-1}. \tag{A2}$$

Differentiating (A1) totally, we find that for the steady states

$$\begin{aligned}&\frac{1}{((1-\beta)e+a(e))^{1-\alpha}}[(1-\beta)+\beta\theta\delta\lambda/(1+e)^{1+\delta}-(1-\alpha)\lambda((1-\beta)+\beta\theta\delta/(1+e)^{1+\delta})-1]de_r\\ &+\frac{1}{((1-\beta)e+a(e))^{1-\alpha}}[\beta\theta\delta(1-\lambda)/(1+e)^{1+\delta}-(1-\alpha)(1-\lambda)((1-\beta)+\beta\theta\delta/(1+e)^{1+\delta})]de_p\\ &=\frac{1}{(1-\beta)e+a(e)}\{[-\lambda Z(e)e-a(e)]de_r+[-(1-\lambda)Z(e)e]de_p\}=0,\end{aligned} \tag{A3}$$

where

$$Z(e) \equiv (1-\alpha)(1-\beta) - \alpha\frac{\beta\theta\delta}{(1+e)^{1+\delta}}.$$

Note that, in the steady states, $e_{rt-1} = e_{pt-1} = e_{at-1} \equiv e$. We use $e = (h(e)\tau)^{\alpha}$ and $h(e)\tau = (1-\beta)e + a(e)$ in the derivation of (A3).

A positive (negative) value of $Z(e)$ means that while an increase in the education level causes the externality to increase the income level, its effect is smaller (greater) than the increase in the education price. Using (A3), we obtain

$$\frac{de_r}{de_p}|_{\Delta e_{rt}=0} = -\frac{(1-\lambda)Z(e)e}{\lambda Z(e)e + a(e)}. \qquad (A4)$$

Differentiating (A2) totally, on the other hand, we find that for the steady states

$$\frac{1}{(1-\beta)e + a(e)}\{[-\lambda Z(e)e]de_r + [-(1-\lambda)Z(e)e - a(e)]\}de_p = 0. \qquad (A5)$$

We then have

$$\frac{de_r}{de_p}|_{\Delta e_{pt}=0} = -\frac{(1-\lambda)Z(e)e + a(e)}{\lambda Z(e)e}. \qquad (A6)$$

Let us show that the relationship noted in (A7) is equivalent to that noted in (A8):

$$\frac{1}{\alpha}e^{1/\alpha-1} \lesseqgtr (1-\beta) + \frac{\beta\theta\delta}{(1+e)^{1+\delta}}. \qquad (A7)$$

$$Z(e)e + a(e) \lesseqgtr 0. \qquad (A8)$$

Equation (A7) implies the relationship between the slopes of $f(e)$ and $g(e)$ that appears in Figure 4.

Using $e^{1/\alpha} = (1-\beta)e + a(e)$, (A7) can be rewritten as

$$(1-\beta)e + a(e) \lesseqgtr \alpha(1-\beta)e + \alpha\frac{\beta\theta\delta}{(1+e)^{1+\delta}}e.$$

This equation implies (A8). The equivalence between (A7) and (A8) implies the following relationships:

$$Z(e^{**})e^{**} + a(e^{**}) < 0 \quad and \quad Z(e^{*})e^{*} + a(e^{*}) > 0. \qquad (A9)$$

Using (A4) and (A6), the difference between the slopes of lines $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ becomes

$$\frac{de_r}{de_p}|_{\Delta e_{rt}=0} - \frac{de_r}{de_p}|_{\Delta e_{pt}=0} = \frac{a(e)[Z(e)e + a(e)]}{\lambda Z(e)e[\lambda Z(e)e + a(e)]}. \qquad (A10)$$

We first consider $a(e^{*}) > 0$ shown in Figure 5. Assuming that $Z(e^{**}) > 0$, this result implies that the slopes represented by (A4) and (A6) take positive values at $e = e^{**}$ because of the presumption of $a(e^{**}) < 0$. Those slopes take negative values at $e = e^{*}$.[17] In addition, the difference between the slopes of lines $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ at $e = e^{**}$ becomes

$$\frac{de_r}{de_p}|_{\Delta e_{rt}=0} < \frac{de_r}{de_p}|_{\Delta e_{pt}=0}.$$

We also have the following relationship at $e = e^{*}$:

$$\frac{de_r}{de_p}|_{\Delta e_{rt}=0} > \frac{de_r}{de_p}|_{\Delta e_{pt}=0}.$$

In the case that $Z(e^{**}) < 0$, on the other hand, the slopes represented by (A4) and (A6) take negative values at $e = e^{**}$. The difference between the slopes of lines $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ written in (A10) becomes positive. However, the dynamics of the education levels of the rich and poor are qualitatively the same as those in the case of $Z(e^{**}) > 0$.

[17] Because $Z(e)$ is an increasing function of $e$, the assumption of $Z(e^{**}) > 0$ implies $Z(e^{*}) > 0$.

We next consider the case of $a(e^*) < 0$ shown in Figure 6. This implies that $Z(e^*) > 0$. The relationship between the slopes represented by (A4) and (A6) at $e = e^{**}$ is the same as that in Figure 5. Assuming that $\lambda Z(e)e + a(e) > 0$ and $(1-\lambda)Z(e)e + a(e) > 0$ at $e = e^*$, those slopes become negative at $e = e^*$.[18] The difference between the slopes of lines $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ at $e = e^*$ becomes

$$\frac{de_r}{de_p}|_{\Delta e_{rt}=0} < \frac{de_r}{de_p}|_{\Delta e_{pt}=0}.$$

When $e_{rt}$ is located inside (outside) $\Delta e_{rt} = 0$, $e_{rt}$ must increase (decrease). Moreover, $e_{pt}$ is located inside (outside) $\Delta e_{pt} = 0$, $e_{pt}$ must increase (decrease).

Last, we check the stability of the steady states. We linearize the system represented by (15) and (16) around the steady states:

$$\frac{\partial e_{rt}}{\partial e_{rt-1}}|_{e_r=e_p=e} = A(e) - \lambda B(e),$$

$$\frac{\partial e_{rt}}{\partial e_{pt-1}}|_{e_r=e_p=e} = -(1-\lambda)B(e),$$

$$\frac{\partial e_{pt}}{\partial e_{rt-1}}|_{e_r=e_p=e} = -\lambda B(e),$$

$$\frac{\partial e_{pt}}{\partial e_{pt-1}}|_{e_r=e_p=e} = A(e) - (1-\lambda)B(e),$$

where

$$A(e) \equiv \frac{1-\beta}{((1-\beta)e + a(e))^{1-\alpha}} \quad and \quad B(e) \equiv \frac{Z(e)}{((1-\beta)e + a(e))^{1-\alpha}}.$$

The implied characteristic polynomial is therefore

$$(\xi - A(e))[\xi - (A(e) - B(e))],$$

[18]Even if those slopes are positive at $e = e^*$, our dynamics do not change qualitatively.

where $\xi$ denotes the eigenvalues.

We first investigate one eigenvalue, $A(e)$. We have

$$A(e) - 1 = \frac{1}{(1-\beta)e + a(e)}[(1-\beta)e - ((1-\beta)e + a(e))] = \frac{1}{(1-\beta)e + a(e)}[-a(e)].$$

This implies the following relationship:

$$A(e) \underset{>}{\overset{<}{-}} 1 \Leftrightarrow a(e) \underset{<}{\overset{>}{-}} 0.$$

In addition, we have $A(e) - B(e) > 0$ because of the equality:

$$A(e) - B(e) = \frac{\alpha[(1-\beta) + \beta\theta\delta/(1+e)^{1+\delta}]}{((1-\beta)e + a(e))^{1-\alpha}}.$$

Furthermore, we have the following relationship because of the equality, $e^{1/\alpha - 1} = ((1-\beta)e + a(e))^{1-\alpha}$:

$$A(e) - B(e) \underset{<}{\overset{>}{-}} 1 \Leftrightarrow \frac{1}{\alpha} e^{1/\alpha - 1} \underset{>}{\overset{<}{-}} (1-\beta) + \frac{\beta\theta\delta}{(1+e)^{1+\delta}}.$$

The right-hand side relationship of this equation shows (A7). Therefore, $A(e) - B(e) > 1$ holds at $e = e^{**}$ and $A(e) - B(e) < 1$ holds at $e = e^{*}$.

Where $a(e^{**}) < 0$, two positive eigenvalues are greater than unity. Where $a(e^{*}) < 0$, while one eigenvalue is greater than unity, the other eigenvalue is less than unity. This implies that while the steady state at $e = e^{**}$ is unstable, the steady state at $e = e^{*}$ is a saddle point. However, where $a(e^{*}) > 0$, we have two positive eigenvalues that are less than unity. That is, the steady state is stable.

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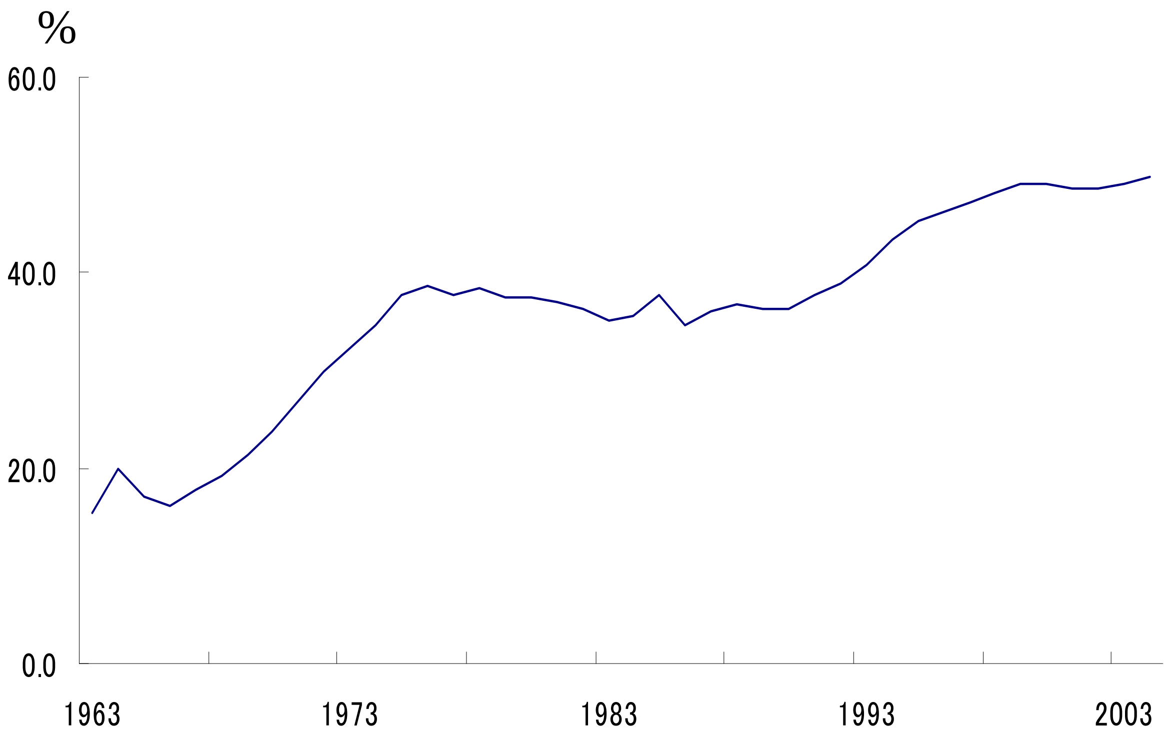


Figure 1. Rate of advancement to universities (including junior colleges)

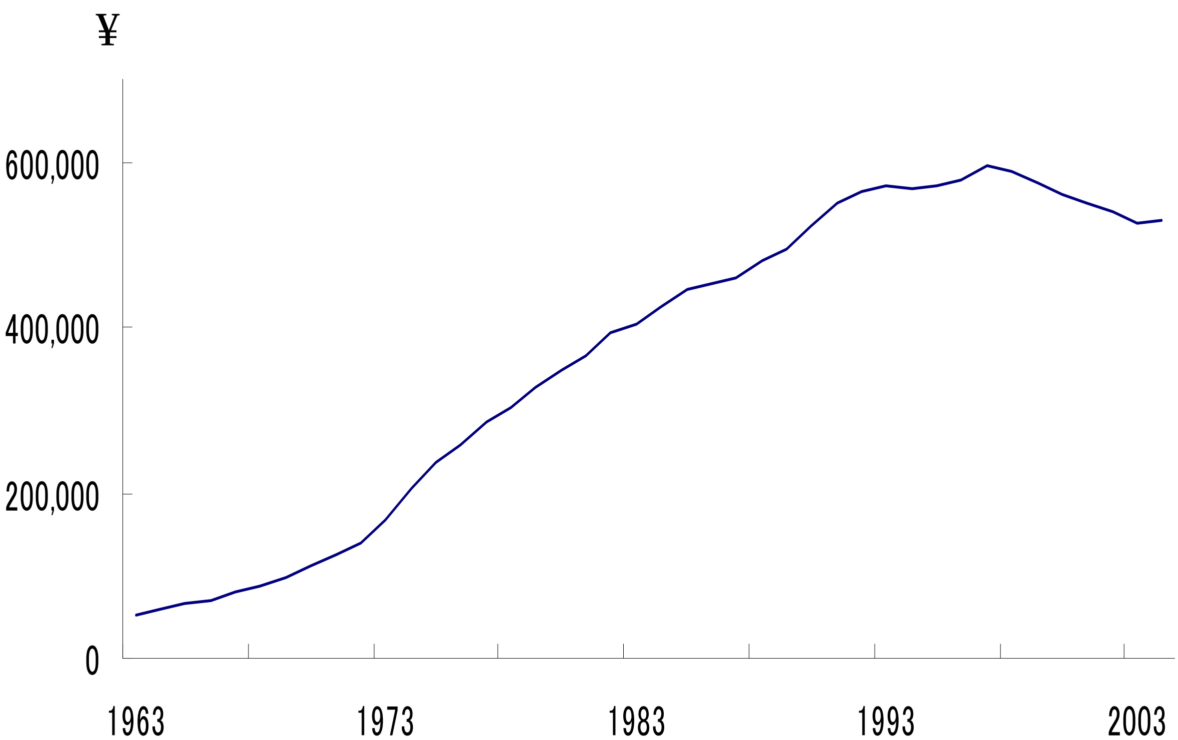


Figure 2. Average annual family income per household

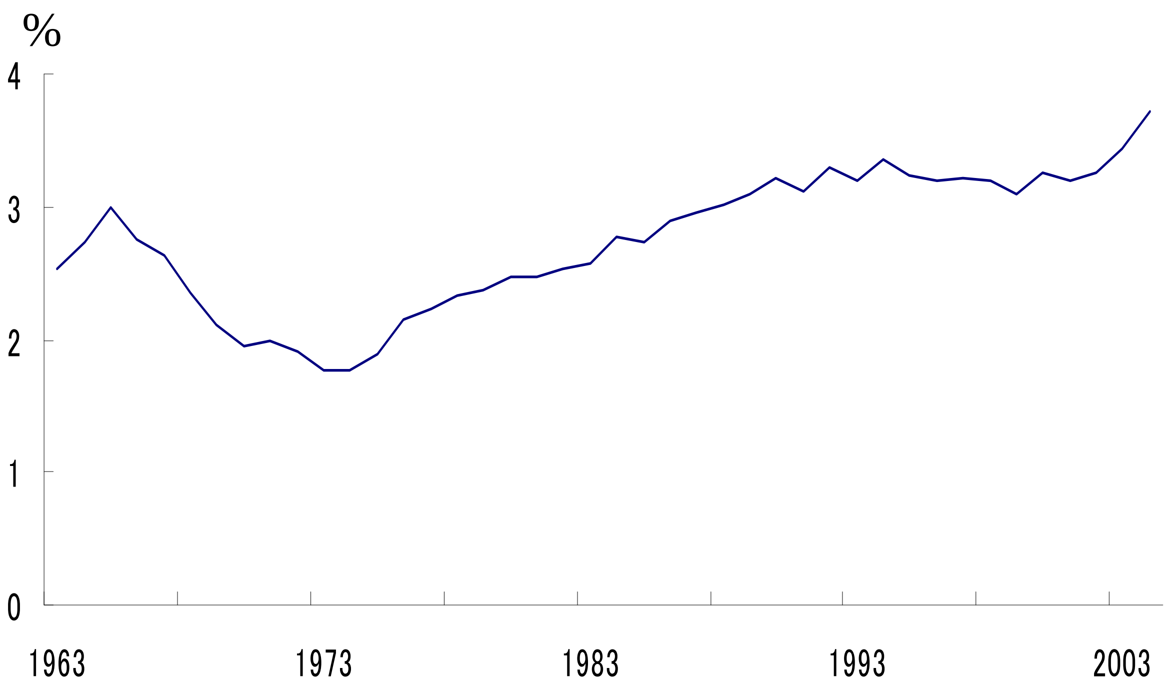


Figure 3. Ratio of educational expenditure to family income

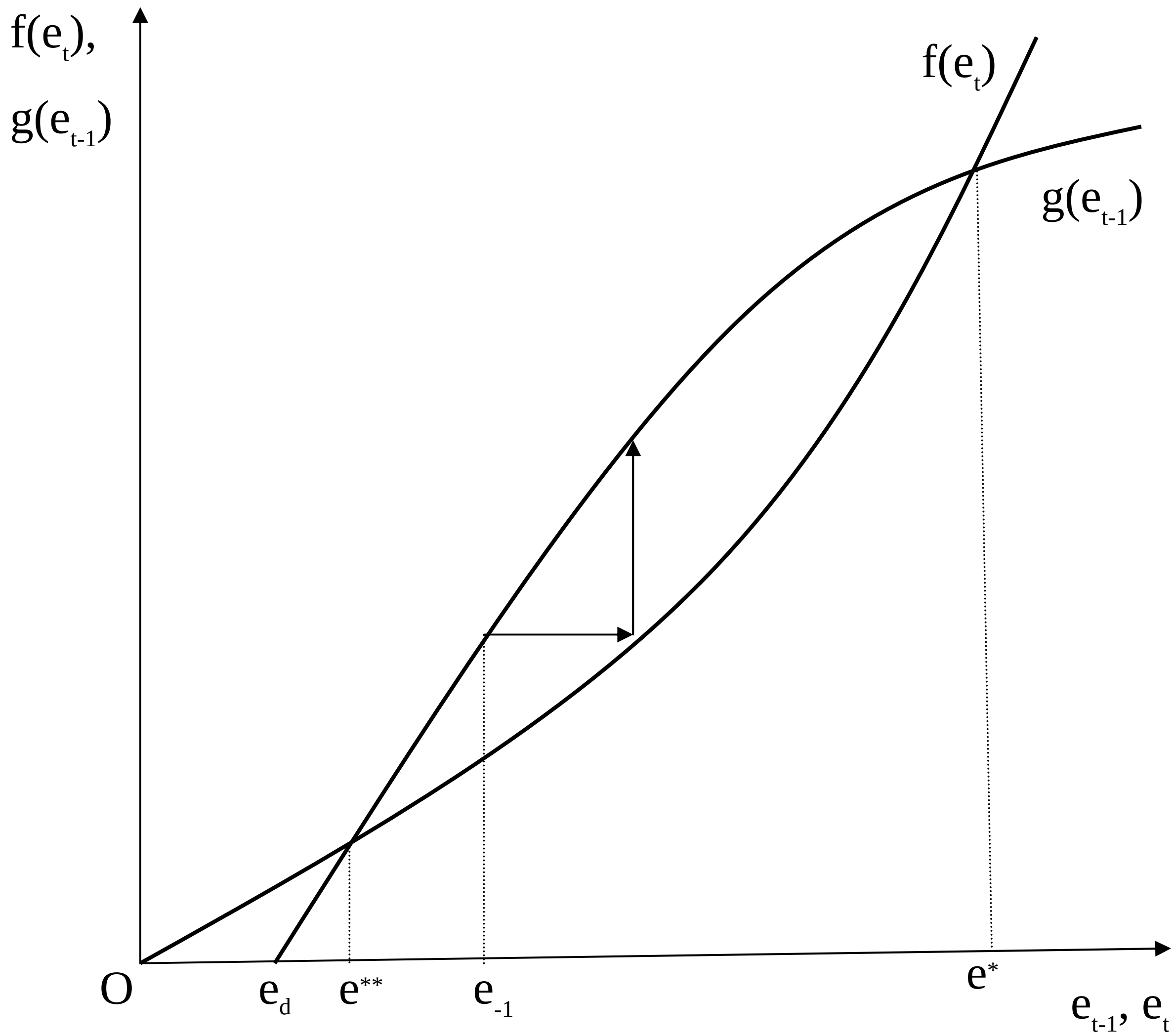


Figure 4. Dynamics of education level

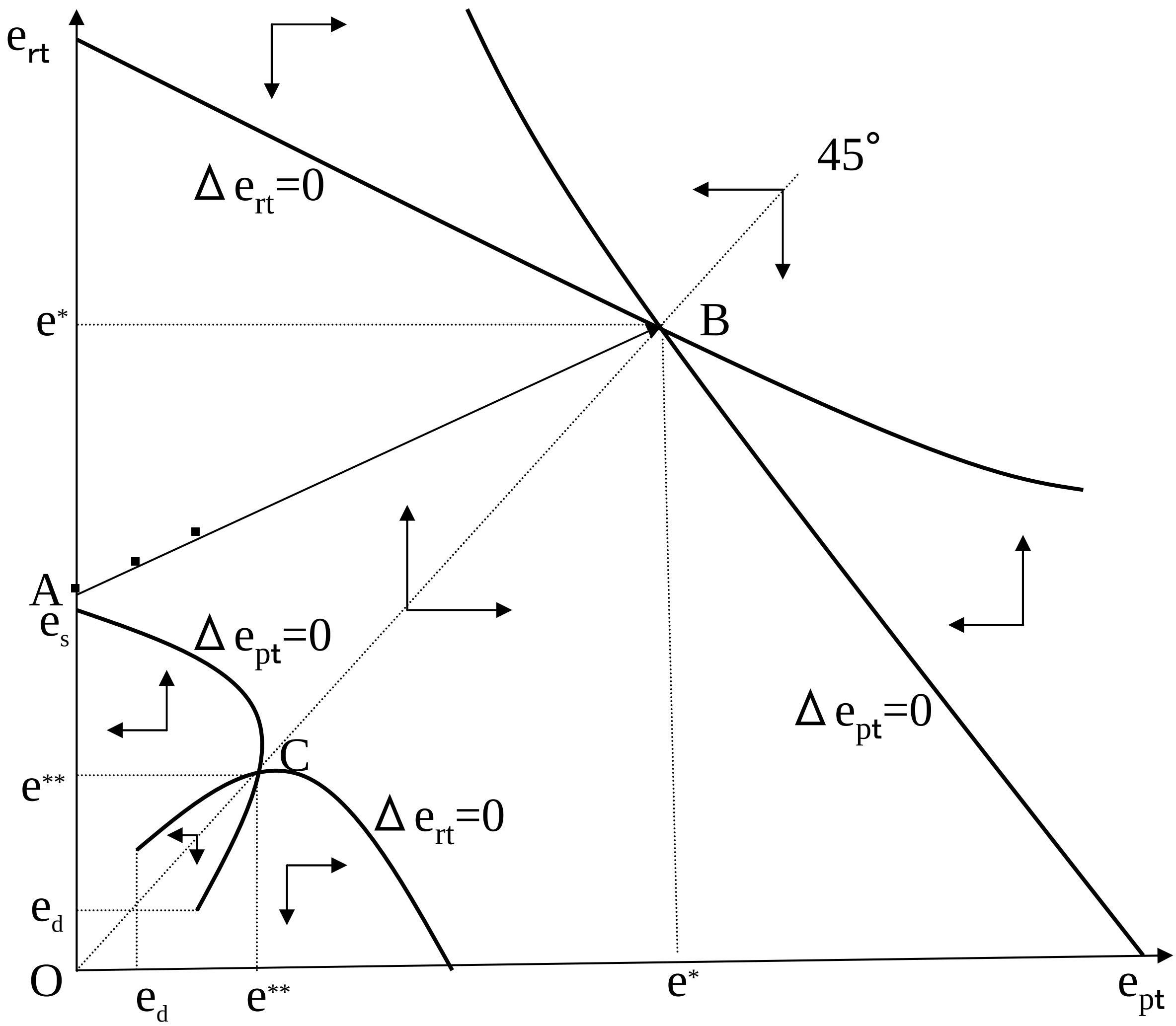


Figure 5. Phase diagram in the case of $a(e^*)>0$

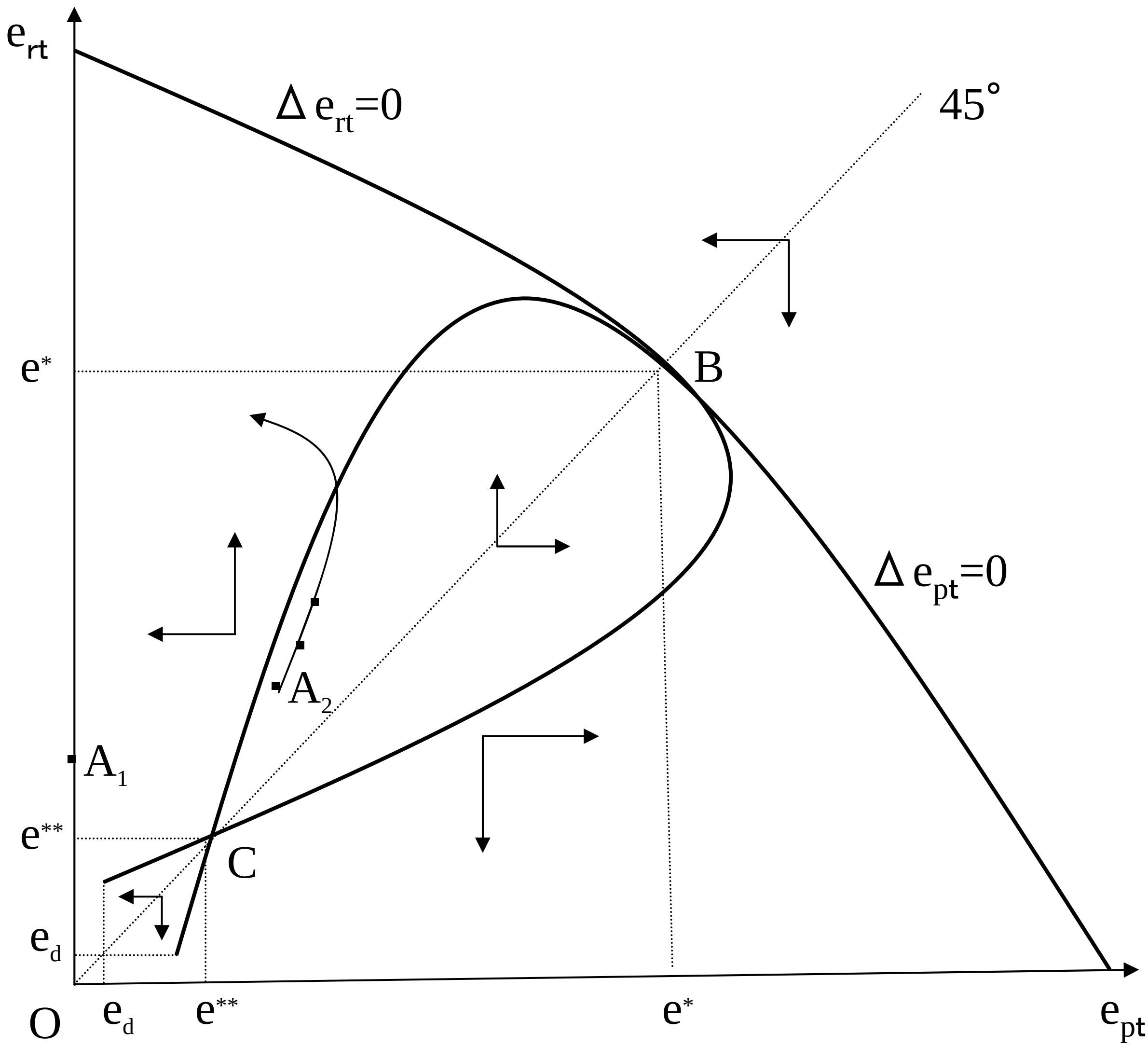


Figure 6. Phase diagram in the case of $a(e^*)<0$