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Significant Effect of Educational Systems on Inequality and Macroeconomic Development

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February 4, 2008
Discussion Paper No. 8

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Significant Effect of Educational Systems on Inequality and Macroeconomic Development*

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Abstract : Assuming borrowing constraints, we investigate how educational systems influence income inequality and macroeconomic development. We consider two educational systems, one in which only a single educational institution exists and the other in which educational institutions are established according to income groups. In the case of the single educational institution, there is a possibility that the poor cannot receive an education because of the high educational price, and therefore, income inequality widens. However, when educational institutions are established according to income group, the poor always receive an education because of the lower educational price and we can reduce income inequality.

Keywords: Educational price, Income inequality, Educational system, Poverty trap;

JEL Classification: I20, O11, O15.

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1. Introduction

How does inequality between the rich and the poor affect macroeconomics? Moreover, how do opportunities for education affect such inequality in the presence of capital market imperfections? Galor and Zeira (1993) authored a pioneering work that investigates the linkage between inequality within in a country and macroeconomics. Assuming an imperfect credit market and indivisibilities in human capital investment, they investigated how the initial distribution of wealth influences inequality and macroeconomic development through individuals' decisions on education. Countries with different initial distributions of wealth clustered around different steady states. Moav (2002) reached the same conclusions as Galor and Zeira (1993) by replacing non-convexities in technology with the convexity of bequest.

The empirical analysis of Quah (1996a,1996b) points out that the income distribution throughout the world has become polarized since World War II. In addition, Persson and Tabellini (1994) and Bénabou (1996a)reported that inequality is negatively related to the rate of economic growth.[1] Galor and Zeira (1993) explained that their implications are consistent with these empirical findings.[2]

[1]Voitchovsky (2005) partly supported this negative relationship between inequality and growth.

[2]By considering human capital investment, Freeman (1996), Owen and Weil (1998), Maoz and Moav (1999), Mookherjee and Ray (2003) and Das (2007) studied the relationship between intergenerational mobility and persistent inequality in the presence of capital market imperfections. Banerjee and Newman (1993), Aghion and Bolton (1997), Piketty (1997), Matsuyama (2000), and Mookherjee and Ray (2002) also showed persistent inequality given an imperfect credit market.

Given the assumptions of credit constraints and a convex bequest function, Galor and Moav (2004) developed a dynamic model explaining both inequality and the process of development. By considering the essential differences between physical capital and human capital, they showed that when the marginal product of capital decreased, the physical capital accumulation was replaced by human capital accumulation as a prime engine of growth. Galor and Moav (2006) emphasized the importance of capital-skill complementarity in the timing of the public education of the masses. They showed that the implications of their model are consistent with empirical facts about education in western countries.

As explained in Galor and Moav (2006), public education of the masses plays an important role for equalization and macroeconomic development in the presence of credit constraints. We further investigate the effect of higher education. How do educational systems of higher education influence income inequality and macroeconomic development? Is it possible to reduce income inequality and attain a high level of GDP by an educational system? We consider two educational systems, one in which only a single educational institution exists and the other in which educational institutions are established according to income group.

Given the assumption of borrowing constraints, we consider that parental preferences depend on consumption and the next period's income of their children. Income is represented by the human capital stock in our model. While the constant term of human capital stock is formed by elementary education, higher education increases the level of human capital. Elementary education that is compulsory is financed by taxes. Higher education is elective and students have to pay tuitions depending on their educational levels. We assume that only the initial educational levels of the rich are different from those of the poor. If the educational price is high compared with the disposable income of the parents, their educational expenditure on their children might become zero. Individuals with no higher education can work as unskilled workers.

A single educational institution tries to meet the demand for education of both the rich and the poor. In this case, there would be a possibility that the poor cannot receive a higher education because of the high educational price. The rich need more teachers than the poor. This implies that the educational price would be beyond the income of the poor. Moreover, even if the poor receive higher education at the beginning, it becomes impossible for them to continue to receive higher education because the educational price increases more than the income of the poor. Therefore, income inequality between the rich and the poor widens in this case. Moav (2002) and Galor and Moav (2004,2006) assumed a convex bequest function so that bequest could take on a value of zero. By assuming a utility function that depends on the next period's income of the child, we show that there are some cases in which the educational price becomes a threshold for a parent's decision on educational expenditure.

If educational institutions are established according to income group, the poor will always receive a higher education. Compared with the case of a single educational institution, the educational price that the poor must pay becomes lower because the demand for education of the poor is smaller than that of the rich. Income inequality will decrease and the GDP level will be larger than in the case of a single educational institution.

We first need to make note of some important points. We consider educational institutions and educational price, whereas most of previous studies on inequality and macroeconomic development did not consider them explicitly. We investigate how educational systems of higher education influence income inequality and macroeconomic development in the presence of borrowing constraints. When the productivity of elementary education is low, the cost of elementary education is high, and the weight of consumption in the utility function is high, in the case of a single educational institution, the poor cannot receive higher education in the long run. Therefore, income inequality widens and GDP remains low. Although increases in efficiency and productivity of higher education increase the educational level and income of the rich, their effect on the average level of education and the GDP would be small because they have no effect on the poor. On the other hand, when the productivity of elementary education is high, the cost of elementary education is low, and the weight of consumption in the utility function is low, it becomes possible for the poor to receive higher education regardless of their income level even in the case of a single educational institution.

If educational institutions are established according to income group, income inequality decreases and GDP becomes large because the poor always can receive a higher education regardless of the productivity and cost of elementary education and the weight of consumption in the utility function. That is, the educational system in which the number of educational institutions sufficiently varies with the number of income groups is Pareto superior to the educational system in which educational institutions are integrated.[3] Therefore, the poor should go to the educational institution for the poor even if the quality of educational institution with respect to the number of teachers was low. Its quality should be improved depending on the income of the poor.

In addition, we show that a poverty trap emerges regardless of educational systems when the efficiency of educational institutions is high and the productivity of higher education is not high. Although educational expenditure is always concave with respect to educational level, educational price becomes concave when efficiency of educational institutions is high. This implies that the dynamics of educational level are not necessarily uniform. Even if educational institutions were established according to income group, there would still be a possibility that income inequalities would remain, depending on the initial educational level of the rich. When both efficiency and productivity of higher education are high in the case that educational institutions are established according to income group, income inequality necessarily disappears with any initial values of the rich and the poor and the level of GDP is

[3]Given the assumptions of local externalities and an economy-wide externality, Bénabou (1996b) investigated the effects of socioeconomic stratification and alternative systems of education finance on inequality and growth. Furthermore, assuming externalities of human capital in communities, Bénabou (1994) and Durlauf (1996) showed persistent inequality caused by stratification or segregation. Our model did not consider any kind of these externalities.

also high. However, even if efficiency and productivity of higher education are high in the case of a single educational institution, unless the productivity of elementary education is high, the cost of elementary education is low, and the weight of consumption in the utility function is low, only the rich will be able to attain education, and therefore, income inequality will widen.

The rest of the paper is organized as follows. Section 2 explains our model. Section 3 investigates two educational systems, one in which only a single educational institution exists and the other in which educational institutions are established according to income group. We present our conclusions in Section 4.

2. Model

We consider a closed overlapping-generations economy. Individuals receive a compulsory education in the first period. Parents have to pay taxes for compulsory education. If parents decide to spend on education, their children will further receive a higher education in the first period. Individuals work in the second period. The disposable income is spent on consumption and higher education. For simplicity, the population in each generation is normalized to unity. We assume that the numbers of rich and poor are respectively, λ and $1 - \lambda$. Their educational levels are denoted as $e_{r,-1}$ and $e_{p,-1}$, respectively. We assume that $e_{r,-1} > 0$ and $e_{p,-1} = 0$.[4] We also consider the case of homogenous individuals, i.e., the case that

[4]The main conclusions do not change even if the educational level of the poor is positive but lower than that of the rich.

$e_{r,-1} = e_{p,-1} > 0$, in Section 2.4. We consider a consumption goods sector and an educational sector of higher education. The educational institution of higher education is run by a non-profit organization. Firms in the consumption goods sector are perfectly competitive.

2.1 Educational sector

We explain an institution for higher education. The workers of the highest educational level can become teachers. That is, teachers are the rich. When the human capital of a teacher is high and the number of teachers is large, education will progress well. Furthermore, there are complementary relationships between teachers and students. Therefore, we presume the following Cobb-Douglas production function:

$$e_t L_t^S = (h(e_{rt-1}) L_t^T)^\alpha (L_t^S)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

where e_t is the educational level to be received in period t. e_{rt-1} is the educational level of teachers, which is also of the rich, $h(e_{rt-1})$ is the human capital stock of teachers, L_t^S is the number of students, and L_t^T the number of teachers, which is also the number of rich employed in the educational sector.[5]

[5]Our model implies the following three cases: all the students in a single educational institution or in the educational institution for the rich are the children of rich, the students in the educational institution for the poor are the children of poor, and the students in a single educational institution are the children of rich and poor. Therefore, in this explanation, the educational level that is received in period $t-1$ is represented simply as e_{t-1}.

There are diminishing returns in the human capital stock of teachers, the numbers of teachers, and the number of students in this production function. Note that diminishing returns in teachers would yield the same results even if we did not consider students as an input. However, for simplicity, we assume homogeneity of degree one in equation (1).[6]

The educational institution is a non-profit organization. Tuitions are used to pay the wages of teachers. In the next section, we show that the wage of a teacher is represented by his or her human capital stock. A balanced budget is written as follows:

$$p_t e_t = h(e_{rt-1})\tau_t, \tag{2}$$

where p_t is the educational price and $\tau_t \equiv L_t^T / L_t^S$ is the number of teachers per student.

While the left-hand side in equation (2) shows the tuition per student, the right-hand side implies the wage cost of teachers per student. The educational price is determined by a zero profit condition. Glomm and Ravikumar (1992), Zhang (1996), Bräuninger and Vidal (2000), and Das (2007) argue the merits of public and private education from the viewpoint of long-term growth. In this paper, we do not consider

[6]Given the assumption of a convex bequest function, Nakajima (2007) explained that wealth inequality cannot be necessarily removed even with perfect credit markets. He used this type of production function to consider the educational price. Rothschild and White (1995) investigated competitive prices and efficient allocations in the case that outputs partially depend on customers as inputs.

any assistance from the government because we would like to investigate the effect of educational price on a parent's decision about education under the assumption of borrowing constraints.

Using equations (1) and (2), the educational price becomes

$$p_t = (h(e_{rt-1})\tau_t)^{1-\alpha}. \tag{3}$$

The educational price is a concave function with respect to the human capital of a teacher and the number of teachers per student. The educational price increases significantly when there are strong diminishing returns in teachers.

Education forms the stock of human capital. For simplicity, human capital is assumed to be of the following linear type:

$$h(e_{it}) = \eta + \gamma e_{it}, \qquad \eta, \gamma > 0, \tag{4}$$

where $i = r, p$. e_{pt} is the educational level of the poor that is received in period t.

The parameter, η, that represents the constant term of human capital stock is obtained by elementary education. Because we assume that elementary education is compulsory, individuals with no higher education can obtain income by working as unskilled workers. The government runs elementary education by collecting taxes. We consider the tax paid per individual, T, that is assumed to be lower than η. This means that the productivity of elementary education is higher than its cost.

2.2 Consumption goods sector

Many competitive firms exist in the consumption goods sector. While the rich are employed both in the educational sector and in the consumption goods sector, the poor are employed only in the consumption goods sector. The production function is assumed to be of the following linear type:

$$Y_t = h(e_{pt-1})(1-\lambda) + h(e_{rt-1})L_t^C, \tag{5}$$

where L_t^C is the number of the rich that is employed in the consumption goods sector. The equality, $L_t^C + L_t^T = \lambda$, holds in the labor market.

Because the price of consumption goods is normalized to be unity, the wage is represented by the level of human capital.

2.3 Individuals

Individuals live in two periods. They receive an elementary education in the first period. Their parents have to pay taxes that are equally levied.[7] Only if parents spend on education, their children receive higher education in the first period. They work in the second period. The disposable income that is equal to the level of human capital is used for consumption and higher education.

We assume that parental preferences depend on consumption and the next period's income of their children.[8] The utility maximization problem of an individual

[7] If we considered taxes that were more heavily levied on the rich, the dynamics would become complicated.

[8] Assuming a convex expenditure on higher education in the presence of borrowing constraints, Nakajima and Nakamura (2008) considered elementary education and higher education. They showed that the productivity of elementary education plays a crucial role for equalization and macroeconomic development.

born in period $t-1$ is as follows:

$$\max_{c_{it}, e_{it}} \beta \ln c_{it} + (1-\beta) \ln h(e_{it}), \tag{6}$$

$$s.t. \qquad h(e_{it-1}) - T = c_{it} + p_t e_{it},$$

where $i = r, p$. c_{rt} and c_{pt} are the consumption levels of the rich and poor, respectively.

A decrease in productivity of elementary education or an increase in cost of elementary education bears on the budget. Using equation (4), the first-order conditions yield the following equations that represent the demand for higher education:

$$p_t e_{it} = (1-\beta)(h(e_{it-1}) - T) - \beta\eta\gamma^{-1} p_t, \qquad i = r, p. \tag{7}$$

The educational price may become a threshold for human capital investment. The threshold is represented by $p_t \beta\eta/((1-\beta)\gamma)$. If the disposable income is not greater than this, educational expenditure becomes zero and the disposable income is entirely spent on consumption.

2.4 Dynamics in the case of homogeneous individuals

While this paper investigates the effect of educational systems on income inequality, we first consider the dynamics of educational level in the case that only homogeneous individuals exist, i.e., that $e_{r,-1} = e_{p,-1}$. Because we assume homogenous individuals, we consider only an educational system in which there exists only a single educational institution.

Using equations (1) and (2), the educational price can be rewritten as

$$p_t = e_t^{(1-\alpha)/\alpha}. \tag{8}$$

While the educational price is a convex function with respect to the educational level in the case that $\alpha \leq 1/2$, it becomes concave in the case that $\alpha > 1/2$.

Using equations (7) and (8), the dynamics of educational level are represented as follows:

$$e_t^{1/\alpha} + \beta\eta\gamma^{-1}e_t^{(1-\alpha)/\alpha} = (1-\beta)(\eta - T) + (1-\beta)\gamma e_{t-1}. \tag{9}$$

Because we assume that the productivity of elementary education is higher than its cost, the educational level and its price are positive regardless of income level. When the income level is low, the educational price becomes low because of the low number of teachers. As a result, the educational price does not act as a threshold for educational expenditure in the case of homogeneous individuals.

The dynamics of educational level crucially differ depending on the efficiency of educational institution and the productivity of higher education. We first investigate the case that $\alpha \leq 1/2$, i.e., that the efficiency of educational institution is low. In this case, both the educational expenditure and the educational price become convex with respect to educational level. Figure 1(a) shows the dynamics of educational level. $f(e_t)$ and $g(e_{t-1})$ represent the left-hand and right-hand sides of equation (9). Given the initial value, e_{-1}, the educational level increases monotonously and converges to e^*. The educational price also increases because of increases in the number of teachers and the educational level of teachers. The number of teachers is represented as

$$\tau_t + \beta\eta\gamma^{-1}h(e_{t-1})^{-\alpha}\tau_t^{1-\alpha} - (1-\beta)\frac{h(e_{t-1}) - T}{h(e_{t-1})} = 0. \tag{10}$$

Let us consider the comparative statistic analysis:

$$\frac{\partial e^*}{\partial \alpha} > 0, \quad \frac{\partial e^*}{\partial \gamma} > 0, \quad \frac{\partial e^*}{\partial \beta} < 0, \quad \frac{\partial e^*}{\partial \eta} \gtreqless 0, \quad \frac{\partial e^*}{\partial T} < 0,$$

where We assume that the educational level of a steady state is greater than unity to include the effect of α.

Any rise in the efficiency of the educational institutions or the productivity of education increases the educational level of the steady state. However, a higher weight of consumption in the utility function decreases the educational level. Because the change in η has two effects, the effect on the educational level is ambiguous. The first is related to the income effect. An increase in the productivity of elementary education increases the demand for higher education because of an increase of parents' income. The second is an effect through the utility function. Because an increase in η also increases the next period's income of children, it plays to decrease the demand for higher education. Thus, the sign of $\partial e^*/\partial\eta$ becomes ambiguous for these two effects.[9] An increase in taxes decreases the educational level because it decreases the disposable income.

Next, in the case of $\alpha > 1/2$, i.e., that the efficiency of the educational institution is high, the educational price becomes concave, even though the educational

[9]Because the former effect dominates the latter effect in the case that equation (23) holds, an increase in the productivity of elementary education increases the educational level.

expenditure is convex with respect to the educational level. Therefore, the educational price increase is smaller than in the case of $\alpha \leq 1/2$. The dynamics of the educational level depend on the productivity of higher education, γ. As shown in Figure 1(b), the educational level monotonously converges to a steady state when γ is low.

Multiple steady states emerge as γ increases. The dynamics are shown in Figure 1(c). While e^{**} is an unstable steady state that implies a poverty trap, both e^{*} and e^{***} are stable. Therefore, if the initial educational level is lower than the educational level of poverty trap as depicted in Figure 1(c), it converges to the low-level steady state, e^{*}. Let us consider the comparative statistic analysis in the case of multiple steady states:

$$\frac{\partial e^{*}}{\partial \alpha}, \frac{\partial e^{***}}{\partial \alpha} > 0, \quad \frac{\partial e^{*}}{\partial \gamma}, \frac{\partial e^{***}}{\partial \gamma} > 0, \quad \frac{\partial e^{*}}{\partial \beta}, \frac{\partial e^{***}}{\partial \beta} < 0, \quad \frac{\partial e^{*}}{\partial \eta}, \frac{\partial e^{***}}{\partial \eta} \gtreqless 0, \quad \frac{\partial e^{*}}{\partial T}, \frac{\partial e^{***}}{\partial T} < 0,$$

$$\frac{\partial e^{**}}{\partial \alpha} < 0, \quad \frac{\partial e^{**}}{\partial \gamma} < 0, \quad \frac{\partial e^{**}}{\partial \beta} > 0 \quad \frac{\partial e^{**}}{\partial \eta} \lesseqgtr 0, \quad \frac{\partial e^{**}}{\partial T} > 0.$$

The results for stable steady states are the same as those for $\alpha \leq 1/2$. However, any rise in the efficiency of the educational institution or the productivity of education decreases the educational level of the unstable steady state. A higher weight of consumption increases the educational level, e^{**}. The effect of η on e^{**} is ambiguous. An increase in T increases e^{**}. When the educational level of a poverty trap decreases, it becomes easier for a trapped economy to converge to the high-level steady state.

If γ is sufficiently high, the poverty trap disappears and only the high-level steady state exists.[10] Figure 1(d) shows the dynamics in this case. Compared with the cases in Figures 1(a), (b) and (c), the one in (d) can attain a much higher educational level, and therefore, the GDP level will also become higher.

3. Educational systems

Here, we assume that the initial educational levels of the poor and rich are zero and positive, respectively. This implies that in the initial period, the poor are unskilled workers and the rich are skilled workers. Given this assumption, we consider a single educational institution in Section 3.1 and educational institutions established according to income group in Section 3.2.

3.1 Single educational institution

When there is only a single educational institution, both rich and poor must go to the same school to receive higher education. The educational institution tries to meet the demands for the rich and poor. The production function of education (equation (1)) can be rewritten as

$$e_{at} = (h(e_{rt-1})\tau_{at})^{\alpha}, \tag{11}$$

where $e_{at} \equiv \lambda e_{rt} + (1-\lambda)e_{pt}$. That is, e_{at} is the average educational level. τ_{at} is the number of teachers per student in the educational institution.

[10] Galor and Weil (2000) and Galor and Moav (2002) explained sustained economic growth from stagnation. In their models, an economy can attain sustained growth because a poverty trap endogenously disappears.

The balanced budget of this educational institution becomes

$$p_{at}e_{at} = h(e_{rt-1})\tau_{at}, \tag{12}$$

where p_{at} is the price of the educational institution.

Using the demand conditions for the educations of the rich and poor, the dynamics of average educational level can be represented as follows:

$$e_{at}^{1/\alpha} + \beta\eta\gamma^{-1}e_{at}^{(1-\alpha)/\alpha} = (1-\beta)(\eta - T) + (1-\beta)\gamma e_{at-1}. \tag{13}$$

We see that the dynamics of average educational level are the same as those of homogenous individuals in equation (9).

The dynamics of education of the rich and poor are respectively

$$e_{rt} = \frac{(1-\beta)(h(e_{rt-1}) - T)}{(h(e_{rt-1})\tau_{at})^{1-\alpha}} - \beta\eta\gamma^{-1}, \tag{14}$$

$$e_{pt} = \frac{(1-\beta)(h(e_{pt-1}) - T)}{(h(e_{rt-1})\tau_{at})^{1-\alpha}} - \beta\eta\gamma^{-1}, \tag{15}$$

where

$$\tau_{at} + \beta\eta\gamma^{-1}h(e_{rt-1})^{-\alpha}\tau_{at}^{1-\alpha} - (1-\beta)\frac{\lambda h(e_{rt-1}) + (1-\lambda)h(e_{pt-1}) - T}{h(e_{rt-1})} = 0. \tag{16}$$

We first consider the case that the following inequality holds:

$$\frac{1-\beta}{\beta}\frac{\gamma(\eta - T)}{\eta} \leq p_a^*, \tag{17}$$

where p_a^* is the educational price that corresponds with the educational level of a steady state, e^*.

Equation (17) implies that unskilled workers cannot spend on education in a steady state. We see the dynamics in Figure 2 that shows the phase diagram for the case of $\alpha \leq 1/2$. While the point, B, is a saddle point, the saddle path exists on the 45 degree line. The dynamics becomes asymmetric depending on the numbers of the rich and poor.

Let us consider equation (17) in detail. Using equation (13), we define the following function:

$$\Omega(e_a) \equiv e_a^{1/\alpha} + \beta\eta\gamma^{-1}e_a^{(1-\alpha)/\alpha} - (1-\beta)(\eta - T) - (1-\beta)\gamma e_a. \tag{18}$$

We can see that $\Omega(e_a = e^*) = 0$.

In addition, we define the educational level as $\hat{e}$ to equate the educational price with the threshold for educational expenditure of unskilled workers:

$$\hat{e}^{(1-\alpha)/\alpha} \equiv \frac{1-\beta}{\beta}\frac{\gamma(\eta - T)}{\eta}. \tag{19}$$

Evaluating the function, $\Omega(e_a)$, with $\hat{e}$, we have the following relationship:

$$\Omega(e_a = \hat{e}) \underset{<}{\overset{>}{-}} 0 \Leftrightarrow \eta(1-\beta) \underset{<}{\overset{>}{-}} T. \tag{20}$$

If the condition, $\Omega(\hat{e}) > 0$, holds, we can see that e^* is lower than $\hat{e}$, and therefore, $(1-\beta)\gamma(\eta - T)/(\beta\eta) > p_a^*$. On the other hand, the condition, $\Omega(\hat{e}) < 0$, implies that $(1-\beta)\gamma(\eta - T)/(\beta\eta) < p_a^*$ because e^* is higher than $\hat{e}$. Therefore, when the productivity of elementary education is low, the cost of elementary education is high, and the weight of consumption in the utility function is high, equation (17) will hold.

Now, we investigate the case that the initial point is represented by A_1 in Figure 2. Because the income of the rich is greater than that of the poor, the demand for education of the rich is larger than that of the poor. Therefore, there is a possibility that the poor cannot spend on education because of the high educational price. The poor cannot receive a higher education when the initial educational level of the rich is higher than the threshold, C. That is, if the inequality in the initial period is large, it will become difficult for the poor to receive a higher education. An increase in the number of rich makes the threshold shift downward. The educational level of the poor remains zero because the educational price increases as the educational level of the rich increases. The students in the educational institution are only the rich. The dynamics of education of the rich are the same as those of homogeneous individuals:

$$e_{rt}^{1/\alpha} + \beta\eta\gamma^{-1}e_{rt}^{(1-\alpha)/\alpha} = (1-\beta)(\eta - T) + (1-\beta)\gamma e_{rt-1}. \tag{21}$$

That is, the dynamics are as in Figures 1(a), (b), (c) and (d).

When the initial point is represented by A_2 that is lower than the threshold, C, the poor also receive an education in the initial period because the initial educational level of the rich is low enough for the poor to make an educational expenditure. There is an equilibrium path above the 45 degree line because the income of the rich is always greater than that of the poor. The educational levels of the rich and poor temporarily increase according to equations (14) and (15), respectively. When the increase in the income of the poor is smaller than the increase in educational price, the educational level of the poor starts to decrease and eventually becomes zero. The educational price rapidly increases in the case that the number of rich is large.

Figure 3 shows the process in which the educational level of the poor decreases.[11] The horizontal and vertical axes represent e_{pt-1} and e_{pt} respectively. The curve, v, moves downward as the educational price increases. The educational level decreases before long, and it eventually becomes zero because of this shift in v. Given that the educational level of the poor is zero, the dynamics of education of the rich follow equation (18).

Therefore, when equation (17) holds, i.e., when the productivity of elementary education is low, the cost of elementary education is high, and the weight of consumption in the utility function is high, a rise in efficiency of the educational institution or productivity of higher education does not influence the poor in the long run whereas it increases the educational level of the rich. This implies that the increase in GDP level is small when the number of poor is large. In addition, if educational assistance does not equalize the educational levels of the poor and rich, the educational level of the poor still converges to zero. The saddle point, B, can be reached only if the initial values of the rich and poor are on the 45 degree line. Therefore, in the case that the productivity of elementary education is low, the cost of elementary education is high, and the weight of consumption in the utility function is high, it will be difficult to reduce income inequality through educational assistance in an economy with a single educational institution.

[11] We assume that $\alpha \leq 1/2$. Where $\alpha > 1/2$, the second derivative of the curve, v, changes from a positive value to a negative one at the same point of inflexion in Figure 1(c).

While Figure 2 shows the case of $\alpha \leq 1/2$, the same conclusion is reached in the other cases. Hence, we have the following proposition for the case that equation (17) holds.

Proposition 1:*If equation (17) holds, the poor cannot receive a higher education in the long run. Only the rich receive a higher education. Therefore, when the educational level of the rich increases from its initial level, income inequality between them widens in the long run.*

Next, we see the case of multiple steady states. We investigate the case that unskilled workers can spend on education in the low-level steady state, but not in a poverty trap:

$$p_a^* < \frac{1-\beta}{\beta}\frac{\gamma(\eta - T)}{\eta} < p_a^{**}. \tag{22}$$

Compared with the case that $(1-\beta)\gamma(\eta-T)/(\beta\eta) < p_a^*$, the productivity of elementary education is high, the cost of elementary education is high, and the weight of consumption in the utility function is low. Note that equation (22) does not require the full educational expenditure in the low-level steady state.

Figure 4 shows the phase diagram in the case that equation (22) holds.[12] The low-level steady state is stable, the poverty trap is unstable, and the high-level steady state is a saddle point. While the condition, $p_a^* < (1-\beta)\gamma(\eta-T)/(\beta\gamma)$, assures that the point, B, is stable, the condition, $(1-\beta)\gamma(\eta-T)/(\beta\gamma) < p_a^{**}$, implies that the point, D, is a saddle point. The slopes of $\Delta e_{rt} = 0$ and $\Delta e_{pt} = 0$ at the point,

[12]Even if we did not assume taxes, there would be a possibility that this case occurs. The low-level steady state would become a saddle point in the case that equation (17) holds.

B, are the opposite of those in Figure 2.

The initial educational level of the rich becomes crucial for the dynamics. If the initial educational level of the rich is lower than the educational level of poverty trap such as that the initial point is represented by A_1, the educational levels of the rich and poor will converge to e^*, and therefore, income inequality will disappear. However, even if the educational level of the poor is higher than the educational level of poverty trap such as that the initial point is represented by A_2, as long as the educational level of the poor is lower than that of the rich, the educational level of the poor will converge to zero because the educational price increases more rapidly than the income of poor. The educational level of the rich will converge to e^{***}. Therefore, income inequality will widen in this case.

Here, we have the following proposition for the case that equation (22) holds.

Proposition 2:*If the educational level of the rich is lower than the educational level of poverty trap in the case that equation (22) holds, the rich and poor always attain the same educational level, and therefore, income inequality between them disappears. However, if the educational level of the rich is higher than the educational level of poverty trap, the educational level of the poor will become zero regardless of their initial value. Only the rich can attain the educational level of the high-level steady state.*

Last, we consider the case that the following inequality holds in the case of a single steady state:

$$p_a^* < \frac{1-\beta}{\beta}\frac{\gamma(\eta - T)}{\eta}. \tag{23}$$

We also assume that $p_a^{***} < (1-\beta)\gamma(\eta - T)/(\beta\gamma)$. This condition excludes the case of multiple steady states.

Figure 5 shows the phase diagram in the case of $\alpha \leq 1/2$. We obtain the same conclusion in the other cases. The initial educational levels of the rich and poor converge to the steady state, B, regardless of the initial point that is represented by A. Therefore, income inequality disappears if the productivity of elementary education is high, the cost of elementary education is high, and the weight of consumption in the utility function is low. Because rises in efficiency and productivity of higher education increase the educational levels of both rich and poor, the increase in GDP level is larger than the case that the poor cannot receive a higher education.

Hence, we have the following proposition for the case that equation (23) holds.

Proposition 3:*If equation (23) holds, the rich and poor can attain the same educational level regardless of the initial values of rich and poor. Therefore, income inequality between them disappears.*

3.2 Educational institutions according to income group

This section considers an educational system where educational institutions are established according to income group. That is, rich and poor go to different schools to receive higher education.

The dynamics of education of the rich follow equation (21). The dynamics of education of the poor are the same as those of homogenous individuals in equation

(9):

$$e_{pt}^{1/\alpha} + \beta\eta\gamma^{-1}e_{pt}^{(1-\alpha)/\alpha} = (1-\beta)(\eta - T) + (1-\beta)\gamma e_{pt-1}. \tag{24}$$

That is, the dynamics of the rich and poor are mutually independent.

The poor necessarily receive a higher education not only regardless of the productivity and cost of elementary education and the weight of consumption in the utility function, but also regardless of their income level. Compared with the case of a single educational institution, the number of teachers of the educational institution where the poor receive a higher education decreases, and therefore, its educational price also decreases. The number of teachers per student in this educational institution, τ_{pt}, is represented as

$$\tau_{pt} + \beta\eta\gamma^{-1}h(e_{rt-1})^{-\alpha}\tau_{pt}^{1-\alpha} - (1-\beta)\frac{h(e_{pt-1}) - T}{h(e_{rt-1})} = 0. \tag{25}$$

τ_{pt} is an increasing function with respect to educational level of the poor. This is the income effect. The educational price of this educational institution does not depend on the educational level of teachers because an increase in the educational level of teachers is balanced with a decrease in the number of teachers. That is, we have $\partial(h(e_{rt-1})\tau_{pt})^{1-\alpha}/\partial e_{rt-1} = 0$.

We can see the dynamics of education of the poor in Figures 1(a),(b),(c) and (d). Given that $e_{p,-1} = 0$, the educational level of the poor converges to e^* in all cases. Although there are some cases that the poor cannot receive education in the case of a single educational institution, the poor can always attain education in the case that educational institutions are established according to income group.

Now, let us consider the effects of efficiency of educational institutions and productivity of higher education on income inequality and GDP. In the case of $\alpha \leq 1/2$, i.e., when efficiency of educational institutions is low, income inequality disappears in the steady state because the educational levels of rich and poor converge to the same value. However, the level of GDP is low. Furthermore, compared with the case that the efficiency of educational institutions is high, the increase in GDP due to the rise in productivity of higher education is small.

In the case of $\alpha > 1/2$, income inequality and GDP level depend on the productivity of higher education. Income inequality disappears when the productivity of higher education is low. However, the GDP level of this case is still low compared with the case that the productivity of higher education is high.

An increase in productivity of higher education causes multiple steady states. When the initial educational level of the rich is lower than the educational level of poverty trap, income inequality will disappear because the educational levels of the rich and poor will converge to the same level, e^*. On the other hand, when the initial educational level of the rich is higher than the educational level of poverty trap, the educational levels of the rich and the poor will converge to e^{***} and e^*, respectively. Therefore, income inequality will remain. If the number of rich is large in this case, the GDP level becomes high because the educational level of the high-level steady state is high. This implication is essentially the same as those of Galor and Zeira (1993) and Moav (2002).

When the productivity of higher education is sufficiently high, income equality is guaranteed. Unlike the other cases, the GDP level becomes high because the educational levels of both of the rich and the poor will converge to e^{***}.

We can see that in the case that equation (23) holds, the educational attainments of the rich and poor become the same between two educational systems, one in which only a single educational institution exists and the other in which educational institutions are established according to income group. However, in the case of a single educational institution, if equation (17) holds or if the educational level of the rich is higher than the educational level of poverty trap in the case that equation (22) holds, only the rich will be able to attain the high educational level and the educational level of the poor will become zero. That is, even if the efficiency and productivity of higher education are high in the case of a single educational institution, low productivity and high cost of elementary education and a high weight of consumption in the utility function would yield income inequality and a low level of GDP.

On the other hand, in the case that educational institutions are established according to income groups, the poor also can receive a higher education not only regardless of their initial educational levels, but also regardless of the productivity and cost of elementary education and the weight of consumption in the utility function. If the efficiency and productivity of higher education are sufficiently high, income equality and a high level of GDP are guaranteed.

Here, we have the following proposition regarding to the effect of educational systems.

Proposition 4: *If educational institutions are established according to income group, not only the rich, but also the poor necessarily receive higher education. Therefore, in the case that either equation (17) or (22) holds, income inequality between them can be reduced and the GDP level becomes high compared to the educational system in which only a single educational institution exists. On the other hand, in the case that equation (23) holds, the educational attainments of individuals are indifferent between those two educational systems.*

We considered two income groups in this paper. Our results would not change even if there were more income groups. If the number of educational institutions did not sufficiently vary with the number of income groups due to the large number of income groups or the small number of educational institutions, income inequality would widen and GDP would not reach a high level because some individuals could not receive education. If there were enough educational institutions to cover all income groups, income inequality would be reduced and GDP level would become high.

4. Conclusion

Assuming borrowing constraints, this paper explored the effect of educational systems on income inequality and macroeconomic development. In the case of a single educational institution, when income inequality in the initial period is large, there is a possibility that the poor cannot receive a higher education because of the high educational price. Furthermore, even if the poor receive a higher education temporarily, the educational level of the poor eventually converges to zero; i.e., income inequality would widen in the long run. Any rise in the efficiency of the educational institution or the productivity of higher education would increase the educational level and the income of only the rich, and therefore, it would yield greater income inequality.

We also showed that if educational institutions are divided up according to income group, the poor will always receive a higher education. This implies that we can reduce income inequality. Compared with the case of a single educational institution, the GDP level will also increase. When the productivity of elementary education is high, the cost of elementary education is low, and the weight of consumption in the utility function is low, even unskilled workers can spend education in a steady state. In this case, our two educational systems become indifferent because the individuals can attain the same levels of education.

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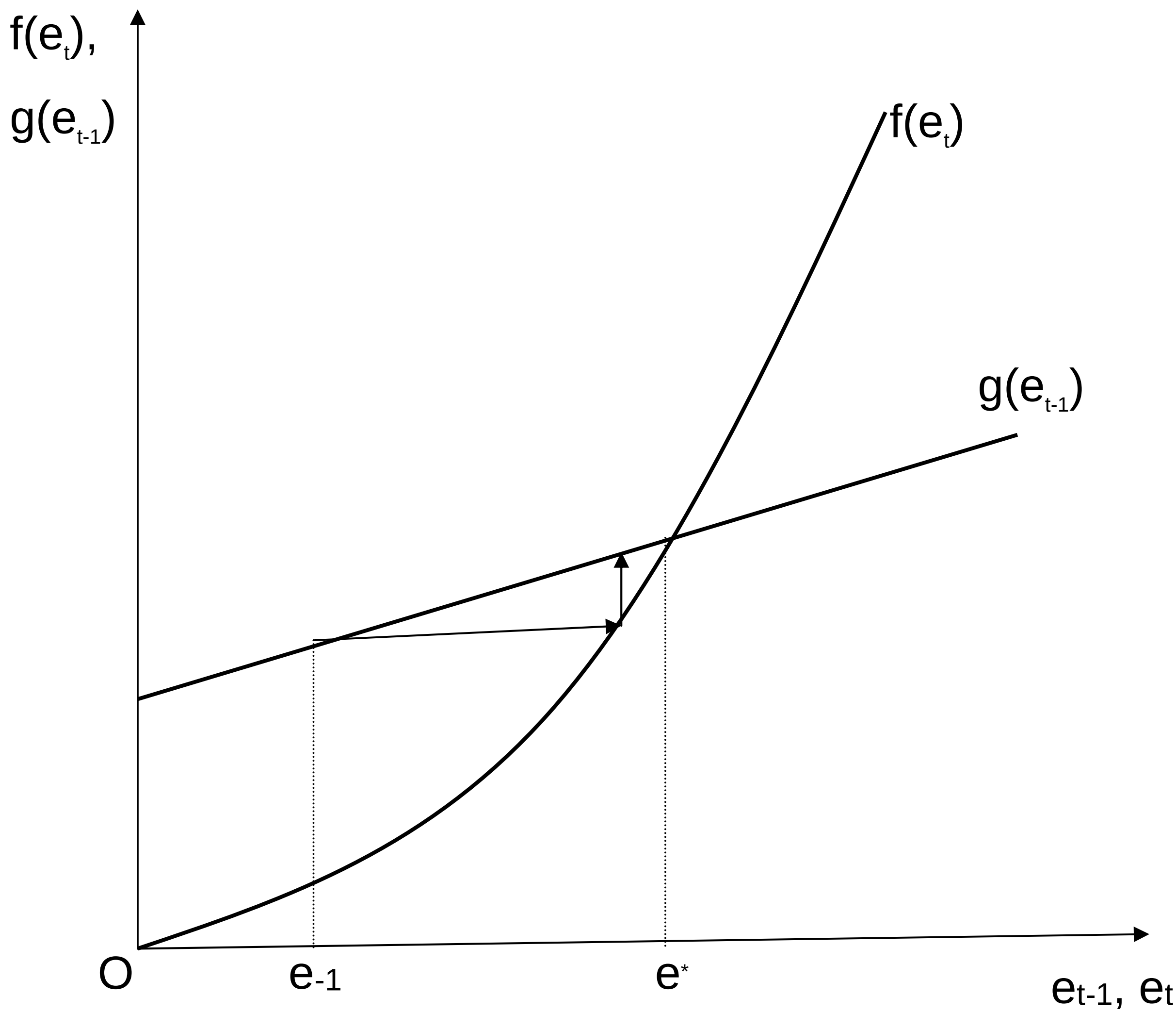

Figure 1 (a). Dynamics of the educational level in the case that $\alpha \leq 1/2$

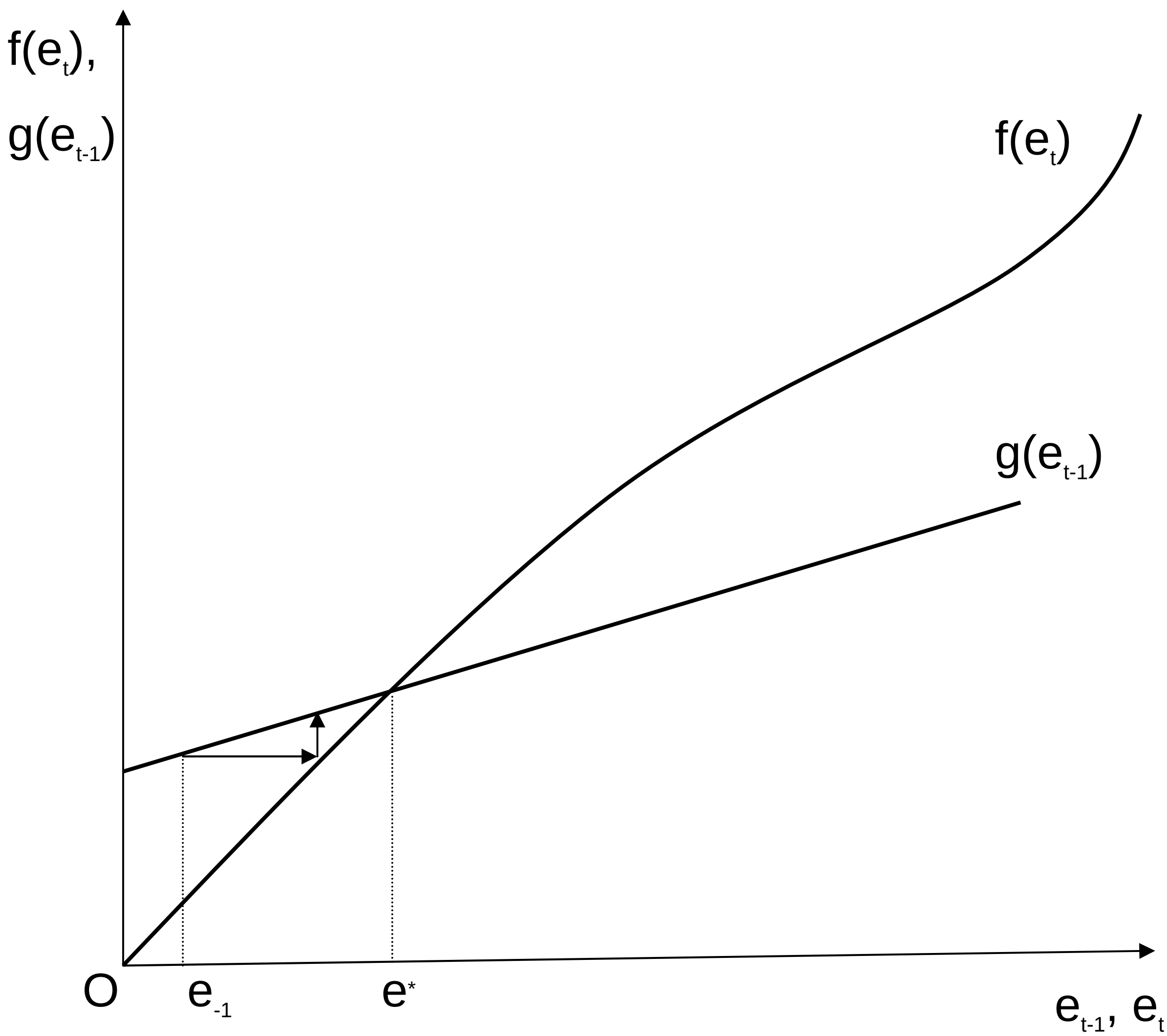

Figure 1 (b). Dynamics of the educational level in the case that α >1/2 and γ is small

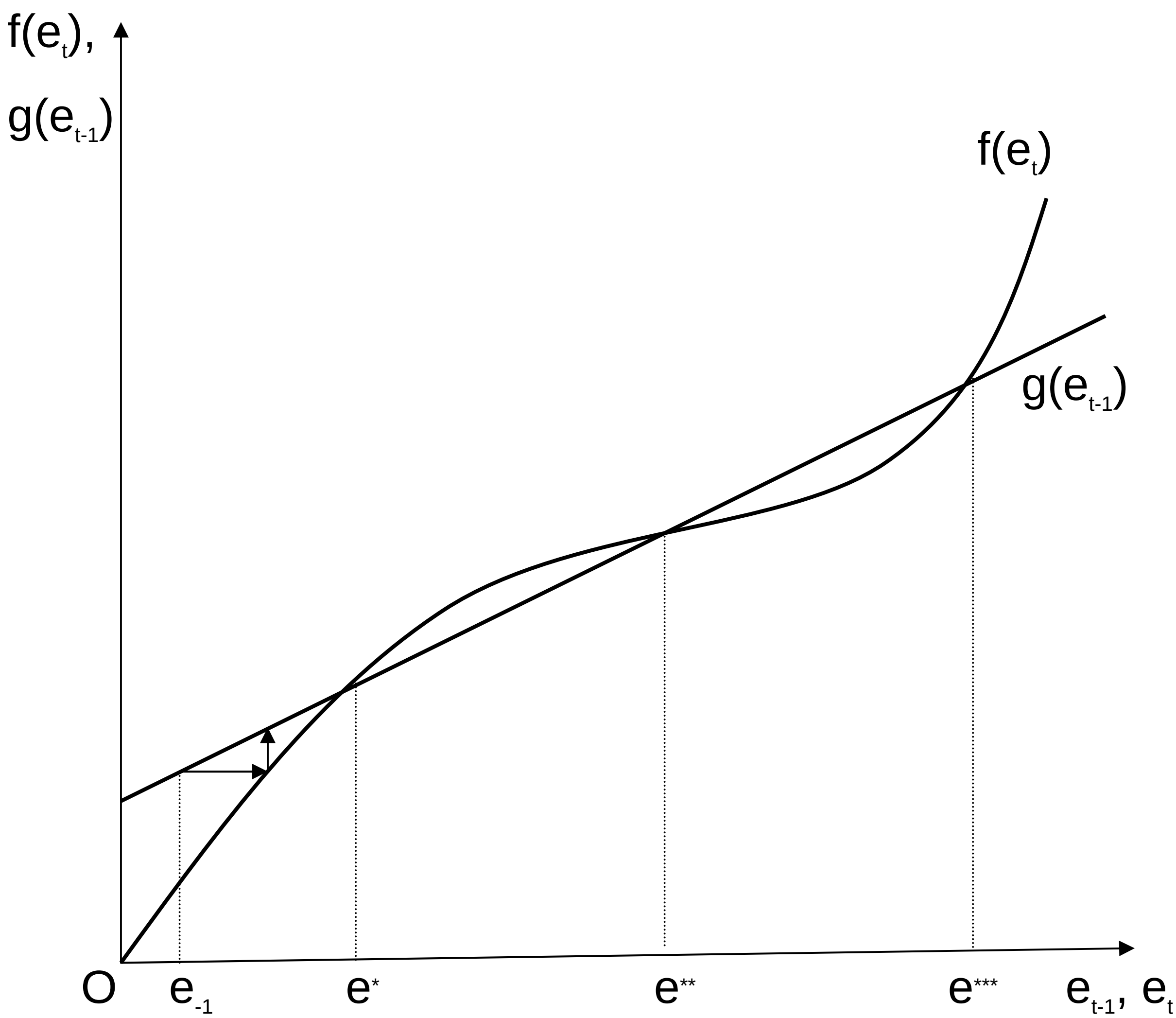

Figure 1 (c). Dynamics of the educational level in the case that α>1/2 and γ is not so large

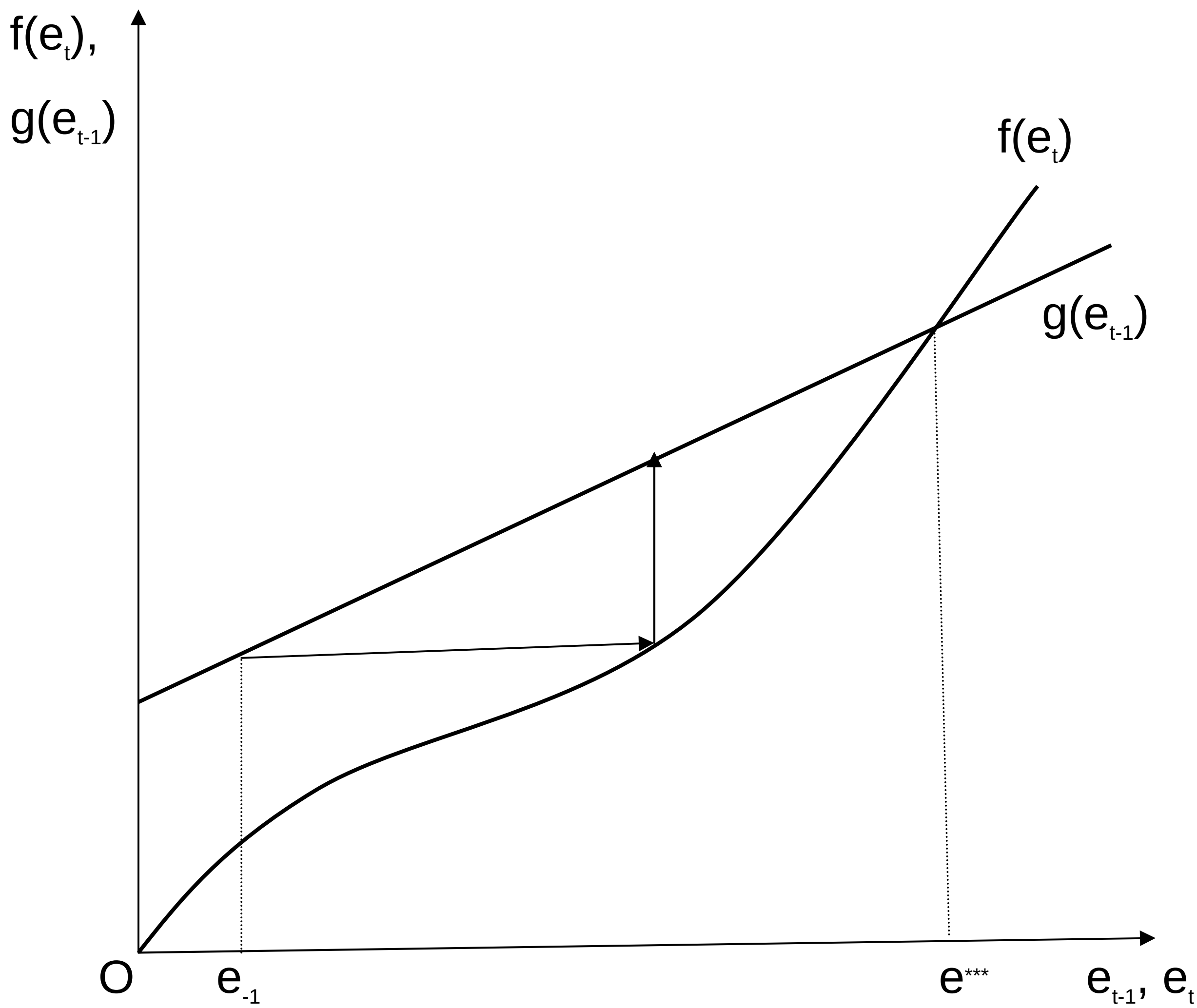

Figure 1 (d). Dynamics of the educational level in the case that α>1/2 and γ is large

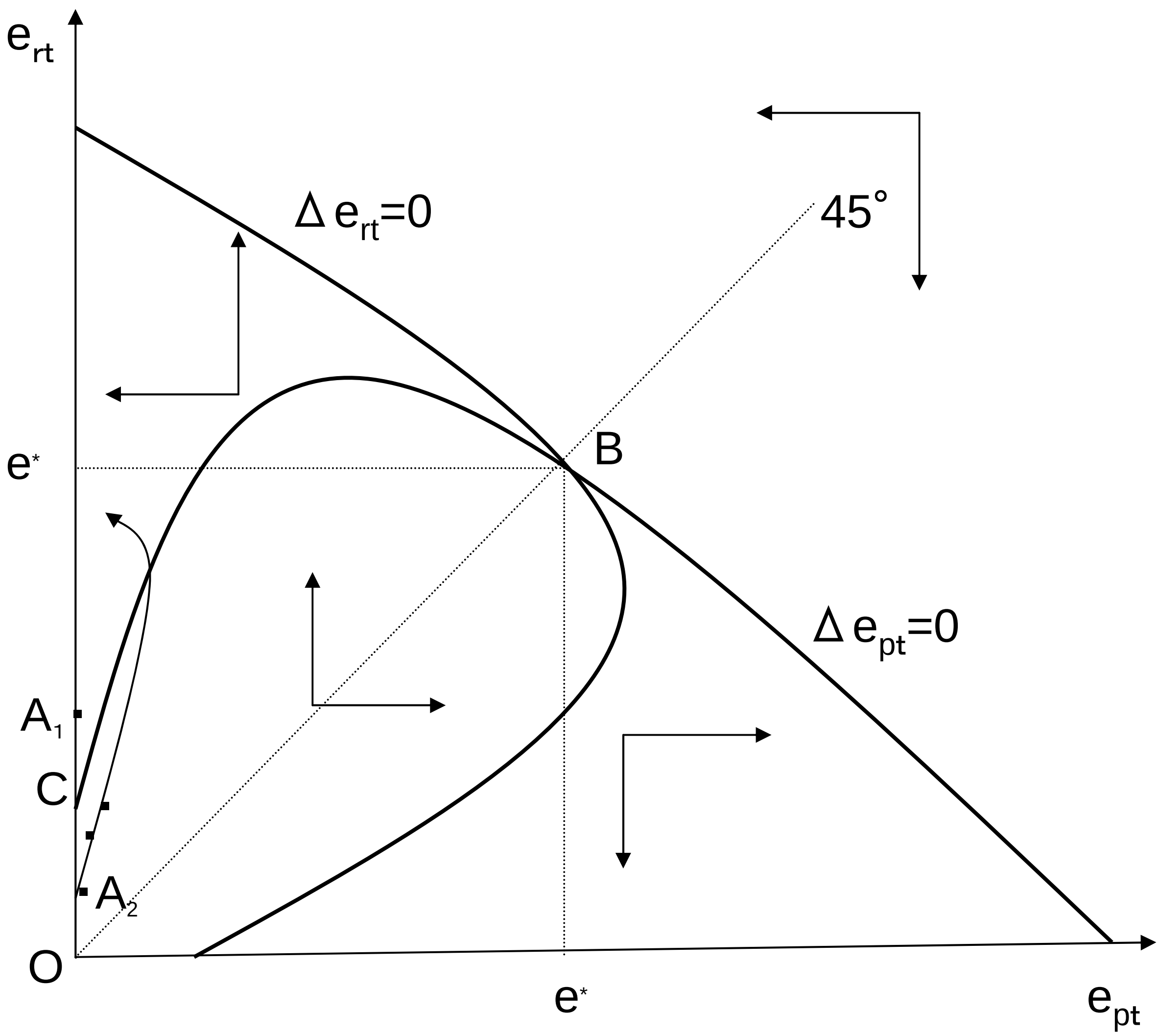

Figure 2. Phase diagram in the case of a single educational institution where equation (17) holds ($\alpha \leq 1/2$)

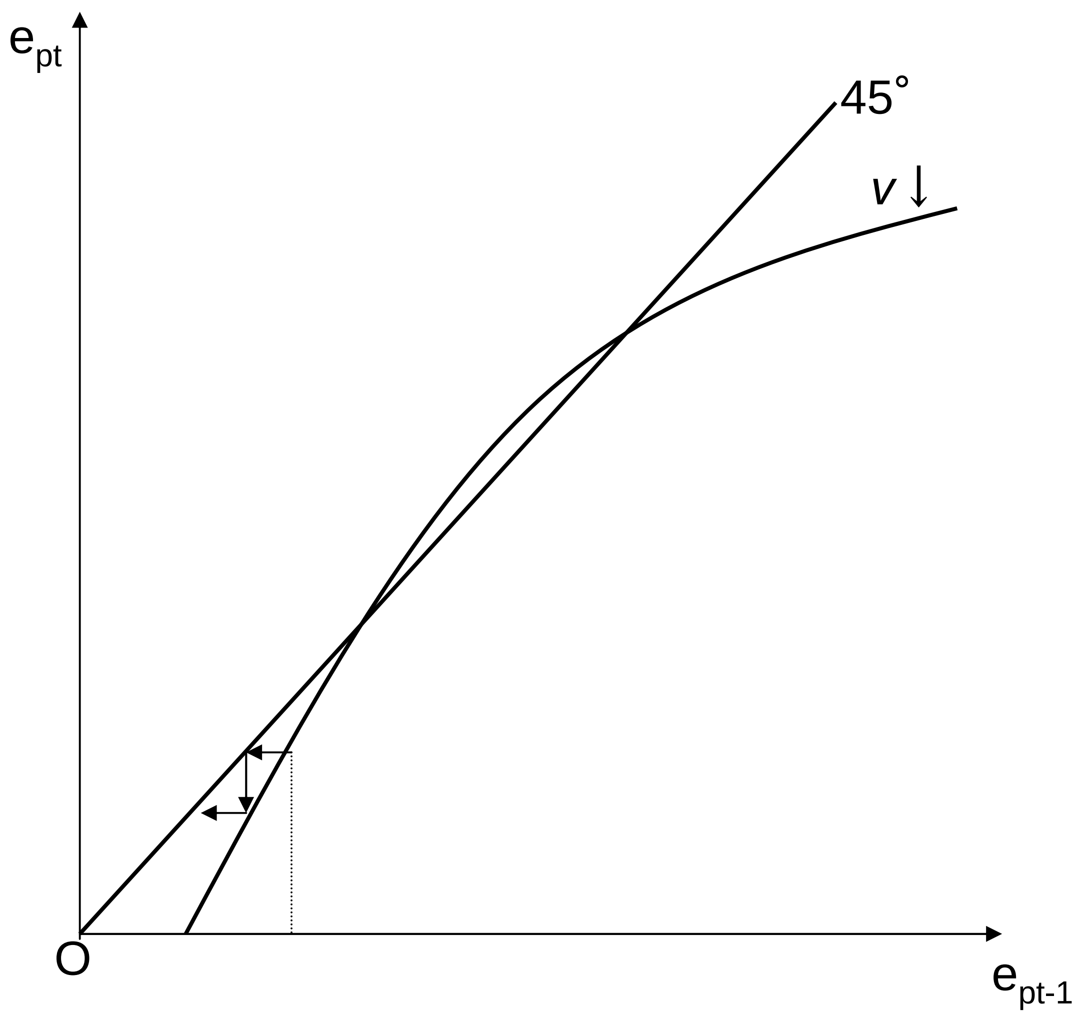

Figure 3. Dynamics of educational level of the poor where equation (17) holds($\alpha \leq 1/2$)

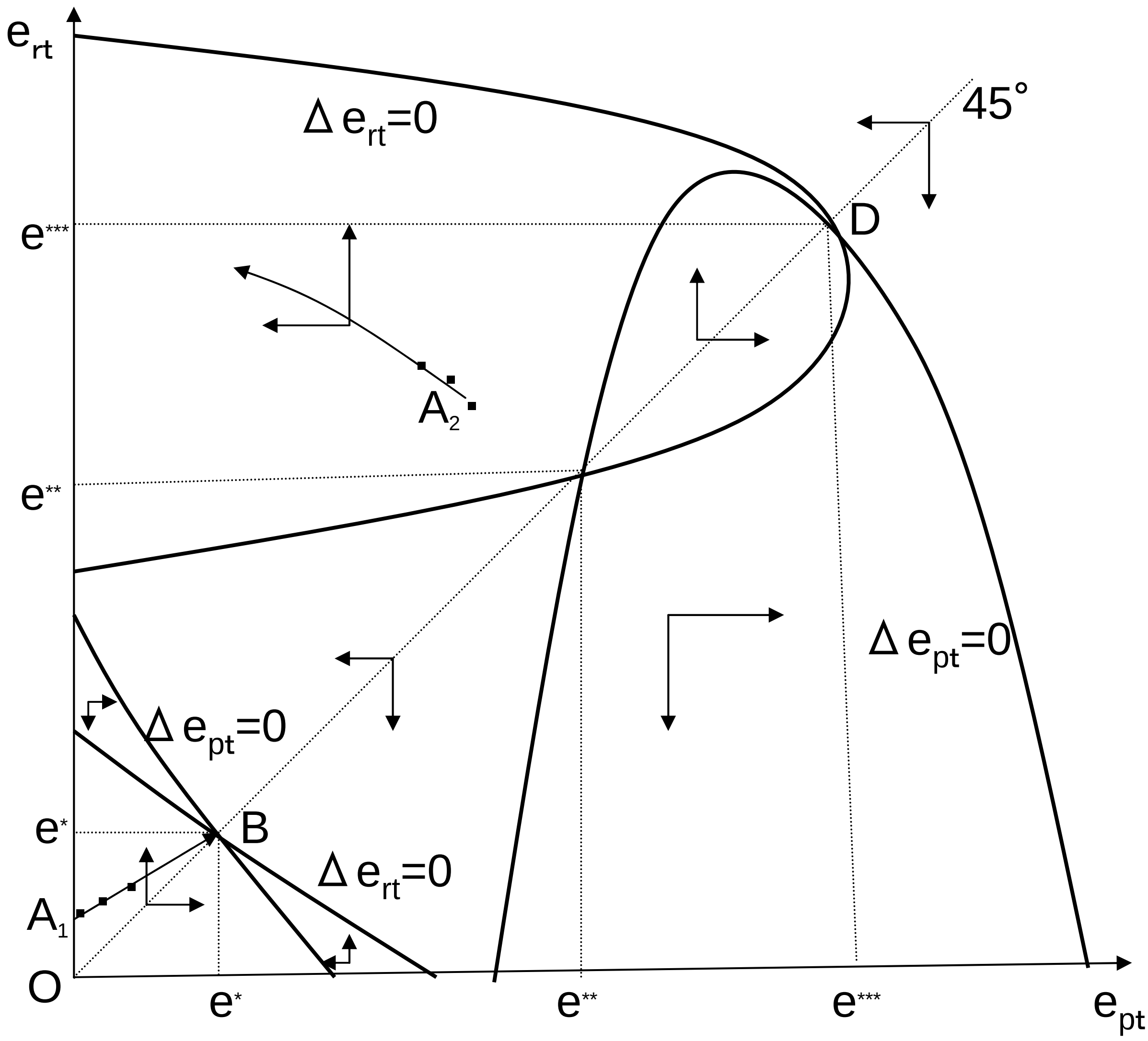

Figure 4. Phase diagram in the case of a single educational institution where equation (22) holds (α>1/2 and γ is not so large)

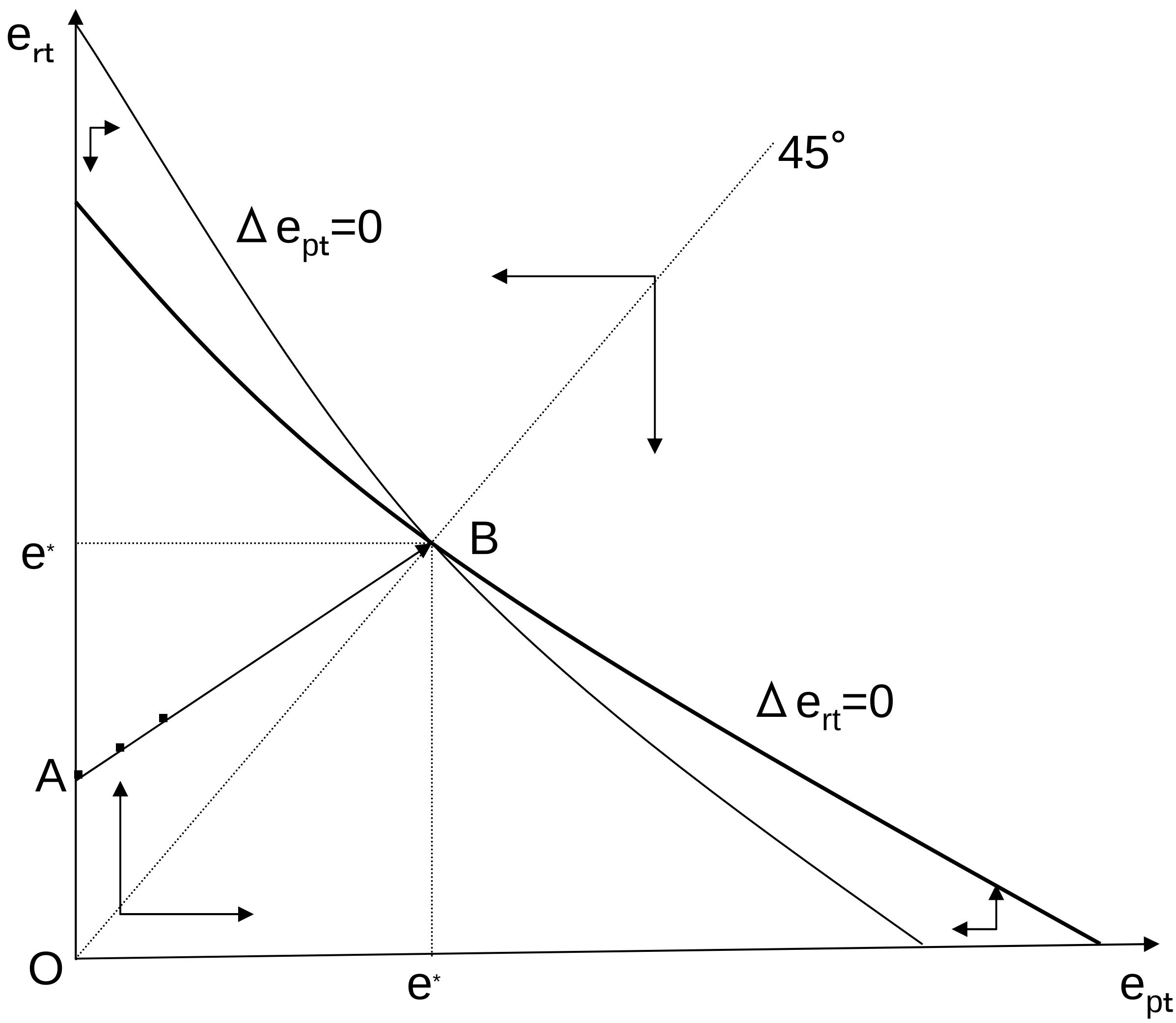

Figure 5. Phase diagram in the case of a single educational institution where equation (23) holds ($\alpha \leq 1/2$)