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Discussion Paper Series

Persistent Inequality in the Framework of a Perfect Loan Market

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September 9, 2006
Discussion Paper No. 1

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Persistent Inequality in the Framework of a Perfect Loan Market

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Abstract

Within the framework of a perfect loan market, this paper clarifies the conditions crucial to whether individual incomes converge or diverge.

Keywords: income inequality, loan market, human capital
JEL classification: D31, I22, J24

1. Introduction

Credit market imperfection has drawn the attention of researchers aiming to explain persistent inequalities. While in developing countries in particular, individuals might be faced with limited access to credit markets, difficulties in borrowing and lending should be less serious in developed countries. Hence, as far as developed economies are concerned, it may be meaningful to consider a perfect credit market and examine how it is that income inequality persists.

The literature on credit market imperfection and inequality includes Benarjee and Newman (1993) and Galor and Zeira (1993) pioneering works, and others.[1] Moav (2002) recently presented a simple model of a small open economy in which income inequality persists due to credit constraints combined with a convex bequest function. While our paper follows Moav with respect to the bequest function, it introduces a new factor into the model: a perfect loan market. The paper then shows a possibility of persistent inequalities, and it clarifies the conditions that determine whether individual

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1 Assuming imperfect credit markets, Benabou (1996), Maoz and Moav (1999), Ghatak and Jiang (2000), Matsuyama (2000), and Mookherjee and Ray (2003) discuss inequality.

incomes converge or diverge.

2. The model

Consider a closed overlapping-generations economy consisting of a goods sector and an education sector.

2.1 Goods production

Competitive firms produce a single type of goods, subject to a production function:

$$Y_t = h(e_{t-1})L_t^Y. \tag{1}$$

The output Y_t at period t is produced using per capita human capital $h(e_{t-1})$ multiplied by the employed labor L_t^Y. Human capital $h(e_{t-1})$ depends on the education at period t-1, e_{t-1}. The function $h(e_{t-1})$ has a property characterized by $h'>0$, $h''<0$, and $h(0)>0$. Since the goods sector is competitive, the real wage per labor will be equal to $h(e_{t-1})$.

2.2 The education sector

Education is an outcome of the collaborations among students and teachers. The total amount of education E_t may be expressed by a function: $E_t = F(L_t^S, h(e_{t-1})L_t^T)$. If the number of students L_t^S and the total human capital of teachers $h(e_{t-1})L_t^T$ become double, E_t will be double as well. Therefore, it may be plausible that the function F is homogeneous of degree one. We can then use a condensed form:

$$e_t = \frac{E_t}{L_t^S} = F\left(1, \frac{L_t^T h(e_{t-1})}{L_t^S}\right) = f(\tau_t h(e_{t-1})). \tag{2}$$

where $\tau_t (= L_t^T / L_t^S)$ denotes the ratio of teachers to students. In addition, it is assumed that the function f is characterized by $f'>0$, $f''<0$, and $f(0)=0$.[2]

Suppose that education (in particular, higher education) is operated by non-profit organizations and that the price of education p_t is determined by a zero profit

[2] An alternative formulation with a constant $\bar{f}$, $e_t = \bar{f}\tau_t h(e_{t-1})$, will not change our results.

condition: $p_t e_t L_t^S - h(e_{t-1})L_t^T = 0$ or equivalently

$$p_t e_t = \tau_t h(e_{t-1}). \tag{3}$$

Since teachers have the same human capital as the other workers, they will earn the same wage $h(e_{t-1})$.

2.3 The households

New households, whose number is *L,* are born every period. Households are identical except for the bequests they receive from their parents. Each Household lives for two periods. In the first period, they decide how much education they receive. As all of them become students, $L_t^S = L$. In the second period, they work and divide income *I* between consumption *c* and bequest *b*. Following Moav (2002), we assume that the behavior of type *i* households in their second period is given by

$$\underset{c_{t+1}^i, b_{t+1}^i}{\text{Max}} \quad \alpha \log c_{t+1}^i + (1-\alpha)\log(\theta + b_{t+1}^i)$$

$$\text{s.t.} \quad I_{t+1}^i = c_{t+1}^i + b_{t+1}^i$$

where $0 < \alpha < 1$, and θ>0. As a result, we obtain

$$b_{t+1}^i = 0 \qquad \text{if } (1-\alpha)I_{t+1}^i \le \alpha\theta, \tag{4}$$

$$b_{t+1}^i = (1-\alpha)I_{t+1}^i - \alpha\theta \qquad \text{if } (1-\alpha)I_{t+1}^i > \alpha\theta. \tag{5}$$

The behavior in their first period is

$$\underset{e_t^i}{\text{Max}} I_{t+1}^i = h(e_t^i) + r_{t+1}(b_t^i - p_t e_t^i) \tag{6}$$

where r_{t+1} denotes a gross interest rate in a loan market. Then the education levels they choose will satisfy:

$$h'(e_t^i) = r_{t+1} p_t. \tag{7}$$

Since the above first order condition is the same for any household, we have $e_t^i = e_t$. Furthermore, let us suppose that there exist two types of households, say type *R* and type *P*, and suppose that $b_t^P < b_t^R$. If $b_t^i - p_t e_t < 0$, the households are borrowers, and if $b_t^i - p_t e_t > 0$, they become lenders.

3. Equilibrium

3.1 Case 1: $b^P > 0$

Let us consider the case where type P households always leave a positive bequest. The loan market equilibrium in period t requires

$$\eta(b_t^R - p_t e_t) + (1-\eta)(b_t^P - p_t e_t) = 0 \tag{8}$$

where $\eta\,(= L^R/L)$ stands for the proportion of type R to the population.[3] Using (5) and (6), the above (8) can be rewritten as

$$\eta\Big[(1-\alpha)\big(h(e_{t-1}) + r_t(b_{t-1}^R - p_{t-1}e_{t-1})\big) - \alpha\theta\Big]$$

$$+(1-\eta)\Big[(1-\alpha)\big(h(e_{t-1}) + r_t(b_{t-1}^P - p_{t-1}e_{t-1})\big) - \alpha\theta\Big] = p_t e_t. \tag{9}$$

Assume loan market clearing in period t-1:

$$\eta(b_{t-1}^R - p_{t-1}e_{t-1}) + (1-\eta)(b_{t-1}^P - p_{t-1}e_{t-1}) = 0, \tag{10}$$

then from (3) and (9) we have

$$(1-\alpha)h(e_{t-1}) - \alpha\theta = \tau_t h(e_{t-1}). \tag{11}$$

Now, from (2) and (11), we obtain a difference equation describing the equilibrium dynamics of e:

$$e_t = f((1-\alpha)h(e_{t-1}) - \alpha\theta). \tag{12}$$

Figure 1 illustrates the behavior of e, assuming the existence of a stationary state e^* and $(1-\alpha)h(0) < \alpha\theta$.[4] If the initial level of education is higher than a threshold $\hat{e}$, e will converge to e^*; which is what we will concentrate on.

3.2 A condition for Case 1 to hold

To simplify analysis, suppose $e_t = e^*$.[5] Then, from (3), (5), (6), (7), and (11), we have the behavior of b^i given by

$$b_{t+1}^i = \frac{(1-\alpha)h'(e^*)e^*}{(1-\alpha)h(e^*) - \alpha\theta} b_t^i + (1-\alpha)(h(e^*) - h'(e^*)e^*) - \alpha\theta. \tag{13}$$

As shown in Figure 2, if

$$(1-\alpha)(h(e^*) - h'(e^*)e^*) - \alpha\theta > 0, \tag{14}$$

then b_t^i converges to b^*, the value of which is

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[3] Once the initial value of η is given, it will be unchanged.

[4] The last inequality corresponds to the assumption (A2) in Moav (2002).

[5] Notice that e autonomously converges to e^*.

$$b^* = (1-\alpha)h(e^*) - \alpha\theta. \tag{15}$$

However, as shown in Figure 3, if

$$(1-\alpha)(h(e^*) - h'(e^*)e^*) - \alpha\theta < 0, \tag{16}$$

b^* is unstable. Since $b_t^P < b_t^R$, sooner or later b^P will be zero.[6] Therefore, the inequality (14) is the condition necessary for Case 1 to hold. It implies that a household who borrows the full amount of educational expenses can still leave a positive bequest.

3.3 Income inequality in Case 1

Consider the situation where $e = e^*$ and $b^P = b^R = b^*$. Then from (3), (6), (11), and (15), it is confirmed that $I^P = I^R = h(e^*)$. Thus, we have a proposition on income inequality:

Proposition 1: If the condition (14) holds, then the initial income gap between type R and type P will disappear in the long run.

3.4 Case 2: $b^P = 0$

Now consider another case where (16) holds and $b^P = 0$. The loan market equilibrium in period *t* is given by

$$\eta(b_t^R - p_t e_t) - (1-\eta)p_t e_t = 0$$

or equivalently

$$\eta b_t^R = p_t e_t. \tag{17}$$

Using (3), (5), (6), (7), and $\eta b_{t-1}^R = p_{t-1}e_{t-1}$, we can rewrite (17) as

$$(1-\alpha)[\eta h(e_{t-1}) + (1-\eta)h'(e_{t-1})e_{t-1}] - \eta\alpha\theta = \tau_t h(e_{t-1}). \tag{18}$$

Accordingly, the dynamics of *e* is indicated by

$$e_t = f\big((1-\alpha)[\eta h(e_{t-1}) + (1-\eta)h'(e_{t-1})e_{t-1}] - \eta\alpha\theta\big). \tag{19}$$

Let us assume that

$$h'(e_{t-1}) + (1-\eta)h''(e_{t-1})e_{t-1} > 0, \tag{20}$$

which implies $\partial I_t^R / \partial e_{t-1} > 0$. Also, assume the existence of a stationary state *e***. The

[6] From (3), (8), (11), and (15), we obtain $\eta b_t^R + (1-\eta)b_t^P = b^*$. Since $b_t^P < b_t^R$, the case of $b_t^P \geq b^*$ is excluded.

stability of e** requires that

$$\left.\frac{de_t}{de_{t-1}}\right|_{e_{t-1}=e^{**}} = f'(e^{**})(1-\alpha)\big(h'(e^{**})+(1-\eta)h''(e^{**})e^{**}\big)<1. \tag{21}$$

Furthermore, in order for b^P to be kept at zero, it is necessary that

$$(1-\alpha)[h(e^{**})-h'(e^{**})e^{**}]-\alpha\theta<0. \tag{22}$$

When (20) ~ (22) hold, the behavior of e in (19) will be similar to that described in Figure 1.

3.5 Income inequality and the effects of wealth distribution in Case 2

From (3), (6), (7), (17) and (18), we have

$$b^R{}^{**}=(1-\alpha)\left(h(e^{**})+\frac{1-\eta}{\eta}h'(e^{**})e^{**}\right)-\alpha\theta, \tag{23}$$

$$I^R{}^{**}=h(e^{**})+\frac{1-\eta}{\eta}h'(e^{**})e^{**}, \tag{24}$$

$$I^P{}^{**}=h(e^{**})-h'(e^{**})e^{**}. \tag{25}$$

Apparently $I^P{}^{**}<I^R{}^{**}$.[7] Offspring of the poor will be persistently poor, and offspring of the rich will be rich.[8] Thus, we have another proposition:

Proposition 2: If (20) ~ (22) hold, the income gap between type R and type P will persist.

Case 2 differs from Case 1 with respect to the effects of a change in η. Taking (20) ~ (22) into account, we have the following results:

$$\frac{de^{**}}{d\eta}<0,\quad \frac{dI^R{}^{**}}{d\eta}<0,\quad \frac{dI^P{}^{**}}{d\eta}<0. \tag{26}$$

Thus, we obtain the last proposition:

Proposition 3: If (20) ~ (22) hold, the concentration of bequests (wealth) in fewer

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[7] Taking (2), (3), (17), and (18) into account, one can confirm the stability of $b^R{}^{**}$ under the condition (21).

[8] The structure that the poor borrow from the rich is in contrast to that of Matsuyama (2000).

households has positive effects on education and individual incomes.

Proposition 3 may be unusual. In Case 2, a decrease in η raises the interest revenue that a type R household receives, and consequently it has positive effects on bequests, provisions of educational loans, human capital, and individual incomes.

4. Concluding remarks

As a cause of persistent inequality, credit market imperfection is deserving of special attention.[9] However, even if an economy has a perfect credit market, income inequality among individuals may last for a long time. In an economy with such inequalities, the offspring of borrowers would be borrowers who have to pay the interest on borrowing, and the offspring of lenders would be lenders who obtain interest revenue.

[9] The empirical study of Clarke et al. (2005) finds that inequality is less when financial development is greater.

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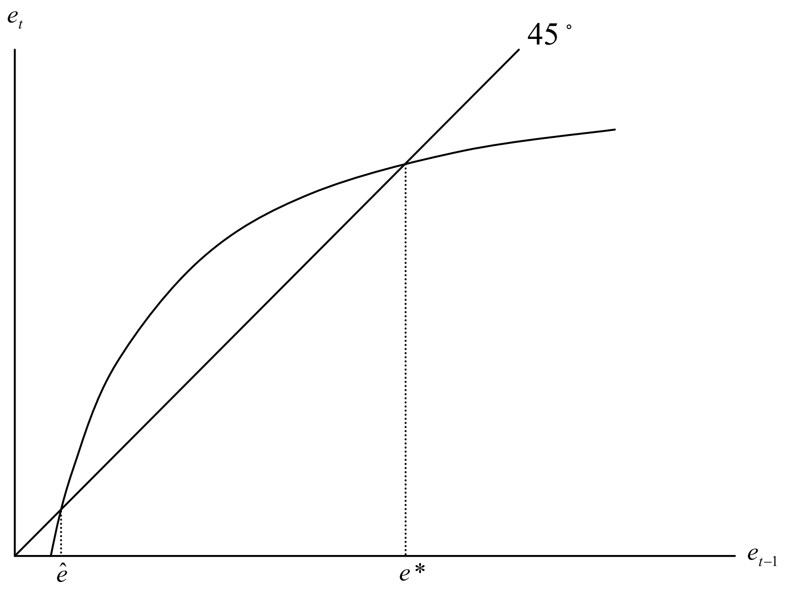

Figure 1. The dynamics of *e*.

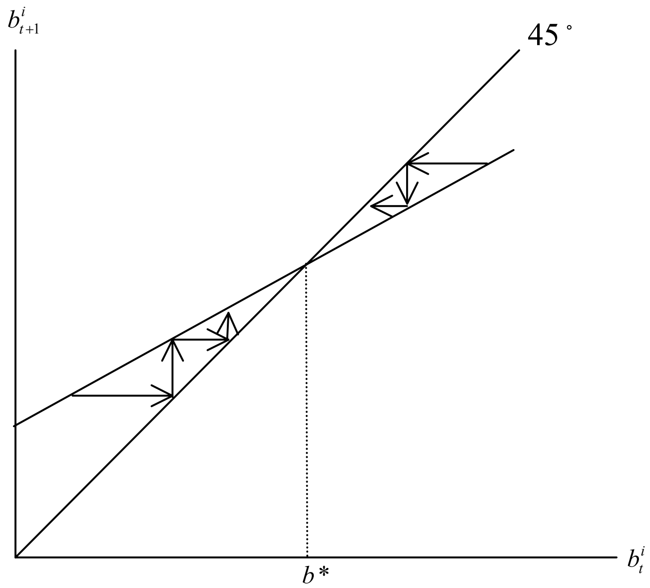

Figure 2. The dynamics of b^i in the case of (14).

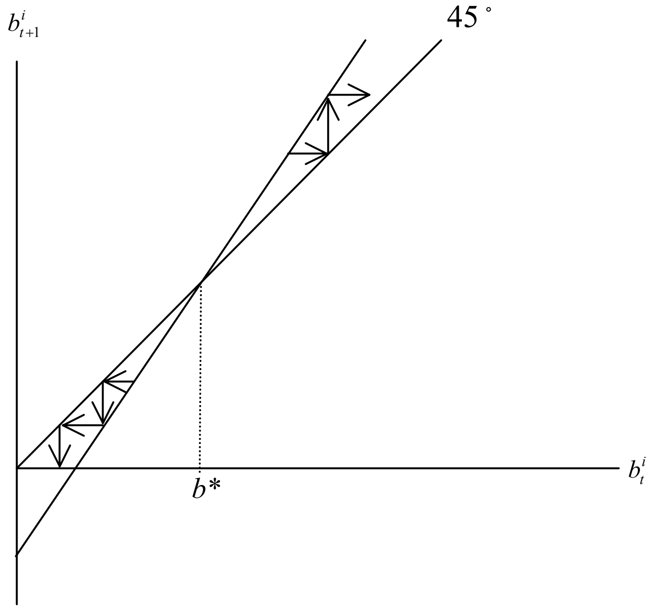

Figure 3. The dynamics of b^i in the case of (16).