Economic Development under Wage Rigidity at the Start of Industrialization: A Note

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Abstract: Assuming wage rigidity, this note sheds light on the role of increasing employment at the start of industrialization. When we describe increases in savings, fertility, and employment, we find a feedback mechanism between the employment rate and bequest level. Thus, focusing on the period of industrialization, this note can complement the unified growth theories in which increases in wages are crucial for economic development.

Keywords: Employment, Savings, Fertility, Wage rigidity, Industrialization.

JEL Classification: E21, E24, J13, O14.
1. Introduction

We observe that in some developed economies, capital per unit of labor accumulates with high savings rates at the start of industrialization, even when the birth rate is high (see the next section). However, there are two points of view with respect to wage flexibility at the start of industrialization, namely, the classical and neoclassical approaches. The classical approach considers the existence of surplus labor and presumes wage rigidity. Conversely, in the neoclassical approach, the marginal productivity of labor determines the wage rate.\(^1\)

As we can observe either increasing wages or increasing employment in the development process, this paper sheds light on the role of increasing employment at the start of industrialization. While we assume an altruistic bequest motive, the relationship between the bequest levels in the current and the next period is nonlinear because of a convex bequest function. Unemployment arises because of the assumption of wage rigidity. The employment rate in the next period depends positively on the bequest level in the current period. Thus, a rise in the bequest level increases the employment rate. The bequest level and fertility rate then increase because the increase in the employment rate increases the income level and the cost of childrearing is assumed to be constant. This implies a feedback mechanism between savings and employment. Multiple steady states then exist because of the nonlinear rela-

\(^1\) Using data on real wages, Fei and Ranis (1964) found a situation of surplus labor in Japan lasting until the WWI boom as real wages changed little during this period (see also Lewis (1958) and Minami (1981)). However, Jorgenson (1966) casted doubt on the presence of surplus labor.
tionship in the dynamics of the employment rate, and these dynamics are essentially the same as those of the bequest level. Accordingly, unless the initial employment rate is particularly low, the employment rate increases, and the economy converges to a stable steady state with low unemployment.

The analysis employs two important assumptions. The first concerns a nonhomothetic utility function that allows for a zero bequest. Assuming a nonhomothetic utility function with an altruistic bequest motive, Moav (2002) and Galor and Moav (2004) derived a Keynesian savings function. We also assume a nonhomothetic utility function after considering endogenous fertility. The second important assumption is wage rigidity. As a result, this analysis sheds light on the role of employment given wage rigidity.

The analysis in this note complements the unified growth theories. For example, Galor and Weil (2000) and Galor and Moav (2004) described the process of economic development with increases in wages. Our analysis attempts to investigate economic development at the start of industrialization in which wages do not depend on the capital-labor ratio. We describe increases in savings, fertility, and employment with the feedback mechanism between savings and employment. Thus, we can show that increases in employment also play an important role in economic development.

2. Empirical features of industrialization

2 See also Nakamura and Nakajima (2011).

3 See Galor (2005) that presented an extensive survey of unified growth theories.
In Japan, industrialization began during the early period of the Meiji era, which lasted from 1868 to 1912. As shown in Table 1, during this time, the production of manufacturing industries increased substantially. Production in light manufacturing industries increased steadily during the Meiji era, whereas production in heavy manufacturing industries increased rapidly only after 1900. Thus, GNP per capita increased steadily and capital per labor unit accumulated because of the high investment ratio (see Kuznets (1971)). As the amount of financial assets held by households increased considerably at this time, the savings rate also increased steadily. The birth rate increased gradually. Real wages changed little from 1900 to 1909. Furthermore, as compared to the period after WWII, the increase in real wages was small from 1910 to 1930. The average ratio of employees to job applicants for general workers between 1921 and 1929 was 33.07%; the equivalent ratio for day workers was 85.69%. Thus, unemployment in urban regions may not have been low.

Importantly, these findings may not be country specific. As shown in Table 1, the production of manufacturing industries continued to increase steadily throughout the 19th century. Economic growth rates were high, as were investment ratios and savings rates, and thus capital per labor unit accumulated steadily (see also Kuznets (1971)). The birth rate continued to increase from 1850 to 1870. Compared to the average growth rate of real wages from 1954 to 1974 (2.68%), the growth of real wages from 1864 to 1890 was not high (see also Deane and Cole (1969)).

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4 Although the industrial revolution started in the 1760s in the UK, data on economic growth are only available from the 19th century onward.
Furthermore, in the US, industrialization progressed rapidly from the second half of the 19th century. As shown in Table 1, the production of manufacturing industries increased substantially during the second half of the 19th century and GNP per capita increased steadily. At the same time, investment ratios and savings rates were very high, and thus capital per labor unit accumulated rapidly (see also Kuznets (1971)). Although the birth rate decreased during the second half of the 19th century, it remained relatively high. Real wages did not increase significantly (see also Kindleberger (1967)).

3. Model

Individuals live for two periods. During the first period, parents pay the cost of childrearing. Parents also provide bequests for their children. During the second period, individuals obtain wages from their labor and earn interest on their assets. The utility function of parents depends on their own consumption, the number of children they have, and the bequest levels for their children. The utility-maximization problem of an individual born in period \( t - 1 \) is represented as follows:

\[
\max_{c_t,n_t,b_t} \alpha_1 \ln c_t + \alpha_2 \ln n_t + (1 - \alpha_1 - \alpha_2) \ln(n_t b_t + \theta), \tag{1}
\]

\[s.t. \quad c_t + \eta n_t + n_t b_t = I_t, \tag{2}\]

where we assume that \( 0 < \alpha_1 < 1, 0 < \alpha_2 < 1, \alpha_1 + \alpha_2 < 1, 0 < \eta, \text{ and } 0 < \theta \). \( c_t \) is the consumption level in period \( t \), \( n_t \) is the number of children in period \( t \), \( b_t \) is the bequest level per child in period \( t \), \( I_t \) is the income level of households in period \( t \), and \( \eta \) is the cost of childrearing.
We assume an altruistic bequest motive. However, we allow zero bequests by assuming $\theta$. Furthermore, we assume that the income level is high enough to satisfy

$$(1 - \alpha_1 - \alpha_2)I_t > \theta(\alpha_1 + \alpha_2). \quad (3)$$

It then becomes possible for parents to leave bequests for their children.

The first-order conditions for utility maximization are

$$b_t = \frac{\eta}{\alpha_2}[(1 - \alpha_1 - \alpha_2) - \frac{\theta}{I_t + \theta}], \quad (4)$$

$$n_t = \frac{\alpha_2}{\eta} (I_t + \theta), \quad (5)$$

$$c_t = \alpha_1 (I_t + \theta). \quad (6)$$

The bequest level increases with a rise in the income level because of $\theta$. The fertility rate also increases with the rise in the income level. When the income level increases, the ratio of the total amount of bequests to income increases.

We consider wage rigidity. The income level is represented as

$$I_t = x_t w + rb_{t-1}, \quad (7)$$

where $x_t$ is the employment rate, $w$ is the wage rate, and $r$ is the gross interest rate.\(^5\)

Next, we consider the firms. The production function is

$$Y_t = AK_t^\gamma N_t^{1-\epsilon}, \quad (8)$$

\(^5\)In every family, some members can work and others cannot. For simplicity, we consider the employment rate of a representative individual.
where we assume that $0 < \epsilon < 1$. $K_t$ and $N_t$ are the capital and labor inputs, respectively, in period $t$, and $A$ is the technological level that is exogenously given.

We assume that unemployment occurs because of the constant wage rate:

$$A(1 - \epsilon)\left(\frac{K_t}{L_t}\right)^\epsilon < w,$$

where $L_t$ is the number of individuals born in period $t - 1$ and is equal to the labor force in period $t$.

Eq. (A1) implies that

$$A(1 - \epsilon)k^\epsilon = w,$$

where $k \equiv K_t/N_t$.

The zero-profit condition implies the following interest rate:

$$r = A\epsilon k^{-(1-\epsilon)} = A^{1/\epsilon} \left(\frac{1 - \epsilon}{w}\right)^{(1-\epsilon)/\epsilon}.$$  

Finally, we examine the equilibrium. The capital stock in period $t+1$ is represented by the aggregate savings in period $t$ represented as $S_t$:

$$K_{t+1} = S_t = n_t b_t L_t.$$  

Dividing (11) by $L_{t+1}$, the employment rate is represented as

$$x_{t+1} = \frac{b_t}{k}.$$  

The bequest level, being equal to per capita savings, determines the employment rate in the next period. Furthermore, the employment rate negatively depends on the capital-labor ratio.
From (9), (10), and (12), the income level written in (7) is proportionate to the bequest level:

$$I_t = wx_t + rb_{t-1} = A(\frac{1}{k})^{1-\epsilon}b_{t-1}. \quad (13)$$

We assume that the initial bequest level is high enough to satisfy (3):

$$b_{-1} > A^{-1}k^{1-\epsilon} \frac{\theta(\alpha_1 + \alpha_2)}{1 - \alpha_1 - \alpha_2}. \quad (A2)$$

Using (4), (5), and (13), the savings rate is represented as

$$\frac{S_t}{Y_t} = \frac{n_t b_t}{I_t} = (1 - \alpha_1 - \alpha_2) - \frac{\theta(\alpha_1 + \alpha_2)}{A(\frac{1}{k})^{1-\epsilon}b_{t-1} + \theta}. \quad (14)$$

Thus, a rise in the bequest level increases the savings rate.

Furthermore, using (5) and (13), the fertility rate can be represented as

$$n_t = \frac{\alpha_2}{\eta}[A(\frac{1}{k})^{1-\epsilon}b_{t-1} + \theta]. \quad (15)$$

When the bequest level increases, the fertility rate increases.

From (4) and (13), the dynamics of bequest level can be rewritten as

$$b_t = \frac{\eta}{\alpha_2}[1 - \alpha_1 - \alpha_2] - \frac{\theta}{A(\frac{1}{k})^{1-\epsilon}b_{t-1} + \theta} \equiv f(b_{t-1}). \quad (16)$$

We have $f(0) < 0$, $f'(b_{t-1}) > 0$, and $f''(b_{t-1}) < 0$. Assumption (A2) ensures $f(b_{-1}) > 0$.

**Proposition 1:** Under Assumptions (A1) and (A2), there are multiple steady states in the bequest levels. The high-bequest steady state is stable, and the low-bequest steady state is unstable.
Proof. Let us consider the point of tangency between $f(b_{t-1})$ and the 45-degree line at $\hat{b}$. At this point, we have

$$\hat{b} = f(\hat{b}).$$

(17)

We also have $f'(\hat{b}) = 1$.

We define $D(b) \equiv \frac{f(b)/b}{f'(b)}$. We obtain $D(0) < 0$ and $D'(b) > 0$. Then, there exists $\hat{b}$ such that $D(\hat{b}) = 1$. By using (17), we define $\hat{A}$ at $\hat{b}$. Multiple steady states can then exist as long as $A > \hat{A}$ holds.

How does the bequest level evolve? Figure 1 depicts the dynamics of the bequest level. We assume that the initial bequest level $b_{-1}$ is located between the low- and high-bequest steady states. The bequest level increases monotonically. The income level increases because of the rise in the bequest level. Consequently, the bequest level increases even further. The fertility rate also increases with the increase in the income level. The bequest level eventually converges to the high-bequest steady state, denoted by $b^h$. If the initial bequest level lies below the low-bequest steady state $b'$, the bequest level declines. The income level decreases with the decline in the bequest level. Consequently, the bequest level decreases even further. The fertility rate also decreases with the decrease in the income level.$^6$

Next, how does the employment rate evolve? From (12) and (16), the dynamics

$^6$ A low technological level implies a low level for the high-bequest steady state and a high level for the low-bequest steady state. Thus, it would be difficult for an economy with a low rate of technological progress to take off.
of the employment rate can be represented as

\[ x_{t+1} = \frac{\eta}{\alpha_2 \kappa} \left[ (1 - \alpha_2 - \alpha_3) - \frac{\theta}{Ak^t x_t + \theta} \right] \equiv g(x_t). \quad (18) \]

We have \( g(0) < 0, g'(x_t) > 0, \) and \( g''(x_t) < 0. \) Assumption \((A2)\) ensures \( g(x_0) > 0. \)

We assume the following inequality that assures the existence of the high-employment steady state:

\[ \frac{\psi^h}{k} < 1. \quad (A3) \]

**Proposition 2:** Under Assumptions \((A1)-(A3),\) there are multiple steady states in the employment rates. The high-employment steady state is stable, and the low-employment steady state is unstable.

Figure 2 depicts the dynamics of the employment rate. We assume that the initial employment rate \( x_0 \) is located between the low- and high-employment steady states. The employment rate rises monotonically. The income level increases because of the rise in the employment rate. Because the bequest level increases with the income level, the employment rate increases even further. The fertility rate also increases with the employment rate. Then, there exists a feedback mechanism between the bequest level and the employment rate. The employment rate eventually converges to the high-employment steady state, denoted as \( x^h. \)

**Appendix: Discussion of wage rigidity**

While a decline in the wage rate decreases the bequest level, it also decreases the capital-labor ratio. When the decrease in the capital-labor ratio outweighs the decrease in the bequest level, the stable employment rate can increase.
In this appendix, we attempt to incorporate a rural region in our model, which focuses on an urban region. When the expected incomes earned in the urban region are higher than the incomes earned in the rural region, individuals living in the rural region attempt to move to the urban region. We assume that the expected wage incomes in the urban region are initially higher than in the rural region:

\[ w_r < wx_l, \]

where \( w_r \) is the wage rate in the rural region.

Given \( w_r \) and \( x_0 \) (\( x_0 > x_l \)), (19) implies downward rigidity in the urban wages. We consider the costs of movement from the rural to the urban regions. These costs differ among individuals living in the rural region because of the different distances between the regions. We assume that the moving costs are uniformly distributed over the interval \([0, z]\). We can represent the cost of a member \( i \in [0, 1] \) who initially lives in the rural region as \( iz \). Considering risk-neutral individuals, the threshold member represented as \( a_t \) can be represented as

\[ w_r = wx_t - a_t z. \]

For simplicity, we assume that individuals moving into the urban region manage to obtain assets equal to those of individuals living in the urban region. Savings and fertility in the urban region then remain intact.
References


Table 1. Industrialization data for Japan, the UK, and the US

<table>
<thead>
<tr>
<th></th>
<th>Japan</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rates in the indices of industrial production</td>
<td>4.49(1880 – 1900)</td>
<td>3.40(1821 – 1840)</td>
<td>5.25(1861 – 1880)</td>
</tr>
<tr>
<td>Growth rates of GNP per capita</td>
<td>5.42(1901 – 1920)</td>
<td>3.25(1841 – 1860)</td>
<td>4.87(1881 – 1900)</td>
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<tr>
<td></td>
<td>1.29(1890 – 1910)</td>
<td>1.64(1851 – 1861)</td>
<td>1.88(1869 – 1888)</td>
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<tr>
<td>Investment ratios</td>
<td>1.84(1910 – 1930)</td>
<td>2.14(1861 – 1871)</td>
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</tr>
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<td></td>
<td>10.60(1881 – 1900)</td>
<td>6.05(1854 – 1860)</td>
<td>19.27(1881 – 1900)</td>
</tr>
<tr>
<td></td>
<td>14.66(1901 – 1920)</td>
<td>7.70(1861 – 1880)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.4(1910 – 1920)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Birth rates</td>
<td>31.8(1900)</td>
<td>33.4(1850)</td>
<td>38.3(1870)</td>
</tr>
<tr>
<td></td>
<td>34.0(1910)</td>
<td>34.3(1860)</td>
<td>35.2(1880)</td>
</tr>
<tr>
<td></td>
<td>36.3(1920)</td>
<td>35.2(1870)</td>
<td>31.5(1890)</td>
</tr>
<tr>
<td>Growth rates of real wages</td>
<td>0.16(1900 – 1909)</td>
<td>1.95(1864 – 1889)</td>
<td>2.07(1864 – 1889)</td>
</tr>
<tr>
<td></td>
<td>2.45(1910 – 1930)</td>
<td></td>
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</tr>
</tbody>
</table>

Note: The savings rates are net rates. The birth rates are per thousand. The other figures are the average percentages during the periods. The figures in parentheses are the periods. The data for Japan are from the Japan Statistical Institute (1968) and Minami (1981). The data for the UK are from Mitchell (1980) and Minami (1981). The data for the US are from the US Department of Commerce (1975) and Minami (1981).
Figure 1. Dynamics of bequest level
Figure 2. Dynamics of employment rate